Assignment Costs for Multiple Sensor Track-to-Track Association

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Abstract – *Successful track-to-track data association in a multi-sensor multi-target scenario is predicated on a proper cost function. The optimal cost function for associating two tracks is well established. This paper investigates the performance of cost functions for more than two tracks. The first two cost functions are based on the idea of sequential pairwise association. The next cost function represents a generalized likelihood so that track association becomes a generalized likelihood ratio test (GLRT). A final cost function is derived as the limit of the likelihood when the target state is random and uniformly distributed over a space that approaches infinite support. This cost function turns the data association algorithm into a likelihood ratio test (LRT) that minimizes Bayes error for the case that no a priori information about the target state is available. Monte Carlo simulations demonstrate the advantages of the infinite prior likelihood function.*

Keywords: Track-to-track association, likelihood ratio tests

1 Introduction

The military has recognized the potential advantages of fusing information from disparate sensor systems to achieve better battlefield surveillance. The exchange of target information between different systems requires robust algorithms to associate the tracks between the systems. A number of track association algorithms have been proposed [1, 2, 3]. These algorithms are built upon a cost function that measures how likely tracks from different systems are viewing the same target.

For the scenario of two sensor systems, the two track association cost function is well established [3, 4, 5, 6]. Extension of this cost function for more than two tracks appear in [1, 2]. The sequential 2-D assignment approach in [2] minimizes a cost function that is the sum of the sequence of two track costs. In contrast, the cost function in [1] represents a generalized likelihood so that track association becomes a generalized likelihood ratio test (GLRT). Unfortunately, the GLRT does not necessarily maximize the probability of correct track associations (P_{ca}).

This paper derives a cost function as the limit of the likelihood when the target state is random and uniformly distributed over a space that approaches infinite support. This cost function turns the data association algorithm into a likelihood ratio test (LRT) that minimizes Bayes error for the case that no *a priori* information about the target state is available. In other words, the cost function maximizes P_{ca} when the potential target state can be any point in $\mathcal{R}^M$ where M is the dimension of the state. Therefore, this cost function is referred to as the *infinite prior likelihood* (IPL) cost.

The sequential 2-D assignment algorithm detailed in [7] is not optimal, but the sum of each pairwise cost function is equivalent to the IPL. In a sense, the sequential assignment approach is a suboptimal method that does not properly account for the coupling between each term in the IPL. In contrast, the S-D assignment algorithm can optimize the IPL cost [1]. Finally, it should be noted that all of the cost functions considered in this paper presume that the track state errors are Gaussian. Cost functions for the non-Gaussian error case is beyond the scope of this paper.

This paper is organized as follows. Section 2 sets up the track association problem as a hypothesis test. Then, Section 3 discusses the classical cost function for associating two tracks and proposed extensions of the cost function for associating n tracks. This section also derives the IPL cost. Then, Section 4 illustrates the advantages of this IPL cost via Monte Carlo simulations. Finally, concluding remarks are provided in Section 5.

2 Problem Formulation

This paper tackles the problem of associating tracks from N disparate sensors. The j-th track for the i-th sensor is denoted as $\mathcal{Z}_{j,i}$. Because the trackers under consideration use the Gaussian model for the target state, the track $\mathcal{Z}_{j,i}$ can be equivalently represented by the state estimate $z_{j,i}$ and covariance $\mathbf{R}_{j,i}$. The focus of this paper centers on the analysis of cost function that can be inserted in the S-D assignment algorithm [1]. The cost function evaluates whether or not tracks from different sensors are likely to originate from the same target. The set of N tracks under consideration is denoted as $\{\mathcal{Z}_{f(i),i} \ : \ \text{for } i = 1, \ldots, N\}$, where $f(i)$ is the associated track under consideration for the i-th sensor. To simplify notation for the remainder of the paper, let the association $j = f(i)$ be implicit and remove the index for the track index j.

A track association algorithm must decide whether a set of N tracks are more likely to be represent the same target than another set of N tracks. To this end, the track association problems should be set up as a hypothesis test where the cost of associating N tracks corresponds to the likelihood of the hypothesis that these N tracks represent the same target. For the i-th sensor, the mean estimate z_i is viewed as a track measurement of the target state and the covariance $\mathbf{R}_i$ as the track measurement error. In other words, for the N tracks, the state estimates are modeled as

$$\begin{aligned} z_1 &= s_1 + \eta_1 \\ z_2 &= s_2 + \eta_2 \\ &\vdots \\ z_N &= s_N + \eta_N, \end{aligned}$$

where η_i for $i = 1, \ldots, N$ represents the measurement or estimation error and s_i is the true target state observed by the track from the i-th sensor. These measurement errors are treated as zero mean, uncorrelated white Gaussian noise vectors with covariances $\mathbf{R}_i$. In this paper, the measurement or state errors for two different tracks are treated as independent so that $E\{\eta_i \eta_k^T\} = \mathbf{R}_i \delta_{i,k}$, where $\delta_{i,k}$ is the Kronecker delta function. In general, the track measurement errors are correlated [6] due to the common process noise in the dynamical equation of the tracker. In future work, the implications of track correlations on the cost function for N sensor track association will be investigated.

Under the positive hypothesis H_1, all N trackers are following the same targets so that $s_1 = s_2 = \cdots = s_N = s$. Therefore, the likelihood function for the tracks to be associated are derived from the following hypothesis,

$$\begin{aligned} H_1: \quad z_1 &= s + \eta_1 \\ z_2 &= s + \eta_2 \\ &\vdots \\ z_N &= s + \eta_N. \end{aligned}$$

The true target state s is unknown, and the likelihood function depends on this unknown value. Different cost functions are derived by either treating the state as a deterministic value to be estimated from the tracks or as a random vector. The following sections discusses the different possible cost functions.

3 Cost Functions for Multisensor Multitarget Association

3.1 Two-Sensor Track Association

When $N = 2$ and the two tracks represent the same target, the unknown target state can clearly be removed by computing the difference between the two tracks, i.e., $z_\Delta = z_2 - z_1$. In other words, if H_1 is true, then $z_\Delta \sim N(0, \mathbf{R}_1 + \mathbf{R}_2)$. The likelihood of the zero mean Gaussian test for independent tracks leads to the following cost function,

$$\begin{aligned} c(\mathcal{Z}_1, \mathcal{Z}_2) = (z_2 - z_1)^T (\mathbf{R}_1 + \mathbf{R}_2)^{-1} (z_2 - z_1) \\ + \log(|\mathbf{R}_1 + \mathbf{R}_2|), \end{aligned} \tag{1}$$

which is smaller when the two tracks are more likely to be associated.

3.2 Sequential Sum of Pairwise Costs

The sequential 2-D assignment, e.g, [2], generalizes the assignment for associating tracks from the $N > 2$ sensors by using the cost function,

$$\begin{aligned} c(\mathcal{Z}_1, \ldots, \mathcal{Z}_N) = \sum_{i=1}^{N-1} \Big[\log(|\mathbf{R}_i + \mathbf{R}_{i+1}|) \\ + (z_{i+1} - z_i)^T (\mathbf{R}_i + \mathbf{R}_{i+1})^{-1} (z_{i+1} - z_i) \Big]. \end{aligned} \tag{2}$$

Clearly, the cost function are likely to be smaller when H_1 is true because each term in the summation of (2) is likely to be small. Actually, the sequential 2-D assignment approach minimizes each term in the cost function by a separate 2-D algorithm. It is easy to show this sequential approach is equivalent to the S-D assignment using (2) because the optimization of each term is decoupled from the other terms. As a result, the 2-D assignments can be performed in parallel.

3.3 Sum of All Pairwise Costs

The sequential cost function given by (2) depends on the ordering of the sensors. A natural modification to correct this deficiency is to define a cost function as the sum of all possible pairwise cost, i.e.,

$$\begin{aligned} c(\mathcal{Z}_1, \ldots, \mathcal{Z}_N) = \sum_{i=1}^{N} \sum_{j=i+1}^{N} \big[\log(|\mathbf{R}_i + \mathbf{R}_j|) + \\ (z_i - z_j)^T (\mathbf{R}_i + \mathbf{R}_j)^{-1} (z_i - z_j) \big]. \end{aligned} \tag{3}$$

While this cost function is intuitively pleasing, it is not proportional to the likelihood of H_1. In subsequent discussions, we will refer to the cost function given in (3) as the *sum of all pairwise costs* (SAPC).

3.4 Generalized Likelihood

For the case that $N > 2$, the unknown target state s can not be easily removed to form a likelihood test. If the target state is treated as a deterministic vector, one is forced to use a GLRT as discussed in [1]. This subsection reviews the derivation of the resulting cost function. The state estimate from the i-th tracker is mean s with covariance $\mathbf{R}_i$, i.e., $z_i \sim N(s, \mathbf{R}_i)$. Because the measurement or state error vectors are assumed independent between tracks, the likelihood for H_1 is

$$\begin{aligned} p(z_1, \ldots, z_N | H_1, s) = \frac{1}{(2\pi)^{\frac{MN}{2}}} \frac{1}{\prod_{i=1}^N |\mathbf{R}_i|^{\frac{1}{2}}} \cdot \\ \cdot \exp\left(-\frac{1}{2} \sum_{i=1}^{N} (z_i - s)^T \mathbf{R}_i^{-1} (z_i - s) \right), \end{aligned}$$

where M is the dimension of the target state. Since s is unknown, a generalized likelihood function with the target state represented by its maximum likelihood (ML) estimate given the N observations (*i.e.,* track estimates) is used. It is easy to show that the ML estimate is given by

$$\hat{s}_{\mathrm{ML}} = \mathbf{R}_{f,1:N} \sum_{i=1}^{N} \mathbf{R}_i^{-1} z_i, \tag{4}$$

where

$$\mathbf{R}_{f,1:N} = \left(\sum_{i=1}^{N} \mathbf{R}_i^{-1} \right)^{-1}. \quad (5)$$

One can interpret the ML state estimate as standard track fusion of the N tracks where $\mathbf{R}_{f,1:N}$ corresponds to the covariance of the fused track. Let the ML state estimate be denoted as

$$z_{f,1:N} = \hat{s}_{\mathrm{ML}}, \quad (6)$$

where $z_{f,1:N}$ represents the track state resulting from fusing tracks $i = 1, \cdots, N$. Then, the generalized likelihood function can be expressed as

$$p(z_1, \ldots, z_N | H_1, s = z_{f,1:N}) = \frac{1}{(2\pi)^{\frac{MN}{2}}} \frac{1}{\prod_{i=1}^{N} |\mathbf{R}_i|^{\frac{1}{2}}} \times \exp\left(-\frac{1}{2} \sum_{i=1}^{N} (z_i - z_{f,1:N})^T \mathbf{R}_i^{-1} (z_i - z_{f,1:N}) \right).$$

By expanding the quadratic form in the exponent and using (4)-(6), the generalized likelihood can be rewritten as

$$p(z_1, \ldots, z_N | H_1, s = z_{f,1:N}) = \frac{1}{(2\pi)^{\frac{MN}{2}}} \frac{1}{\prod_{i=1}^{N} |\mathbf{R}_i|^{\frac{1}{2}}} \times \exp\left(-\frac{1}{2} \left[\sum_{i=1}^{N} z_i^T \mathbf{R}_i^{-1} z_i - z_{f,1:N}^T \mathbf{R}_{f,1:N}^{-1} z_{f,1:N} \right] \right).$$

Finally, the GLRT is equivalent of finding the set of N tracks that minimizes the cost function given by

$$c(\mathcal{Z}_1, \ldots, \mathcal{Z}_N) = \sum_{i=1}^{N} \left[\log(|\mathbf{R}_i|) + z_i^T \mathbf{R}_i^{-1} z_i \right] - z_{f,1:N}^T \mathbf{R}_{f,1:N}^{-1} z_{f,1:N}, \quad (7a)$$

or, equivalently,

$$c(\mathcal{Z}_1, \ldots, \mathcal{Z}_N) = \sum_{i=1}^{N} \left[\log(|\mathbf{R}_i|) + (z_i - z_{f,1:N})^T \mathbf{R}_i^{-1} (z_i - z_{f,1:N}) \right]. \quad (7b)$$

3.5 Infinite Prior Likelihood

At a given instance of time, the target state can be treated as an unknown deterministic value. However, the target state changes randomly, and the value of the state can actually cover a range of values. Therefore, one can model the state as a random value. Then, the likelihood function is the expected value of the likelihood function conditioned on the target state,

$$p(z_1, \ldots, z_N | H_1) = \int_{\mathcal{S}} p(z_1, \ldots, z_N | H_1, s) p(s) ds,$$

where $p(s)$ is the prior probability density function for the state and $\mathcal{S}$ represents all possible values of the target state. In practice, the set $\mathcal{S}$ is determined by the field of view (FOV) of the sensor suite and the physical limitations on the kinematic states of the target, e.g., velocity, acceleration, etc. Without any knowledge of the environment, e.g., terrain, roads, etc., all target states in $\mathcal{S}$ are equally likely. Therefore, $p(s)$ should be treated as uniform over $\mathcal{S}$. Then, the likelihood function simplifies to

$$p(z_1, \ldots, z_N | H_1) = \frac{1}{|\mathcal{S}|} \int_{\mathcal{S}} p(z_1, \ldots, z_N | H_1, s) ds. \quad (8)$$

Unfortunately, a closed form expression for the likelihood function does not in general exist, even for our case that the conditional likelihood is Gaussian. When an expression for the likelihood function does exist, it leads to a cost function that requires computationally expensive nonlinear function calls, e.g., the error function `erf(·)`.

In this paper, the infinite prior likelihood (IPL) is derived to be limit of the likelihood function for an uniform prior as the cardinality of $\mathcal{S}$ goes to infinity. While the likelihood given by (8) is optimal, the IPL leads to a simple expression where the resulting cost function does not require any expensive nonlinear operations. To this end, the likelihood is computed from the prior distribution of state s, which is the M dimensional jointly Gaussian distribution of zero mean and covariance $\mathbf{P}$, i.e., $s \sim N(0, \mathbf{P})$. Then

$$\mathbf{P} = \sigma^2 \mathbf{I}. \quad (9)$$

In the limit as the variance σ^2 go to infinity, all target states become equally likely over $\mathcal{R}^M$. Actually, the likelihood goes to zero because the support of $\mathcal{S}$ is unbounded. For the likelihood to converge to a non-zero value, the likelihood is multiplied by the determinant of $\mathbf{P}$ with a finite σ^2 before the limit of σ^2 goes to infinity. The details of the derivation follow.

Given the prior statistics for the state s, the expected value for the N tracks are zero, i.e., $E\{z_i\} = \vec{0}$ for $i = 1, \ldots, N$. The second order statistics for the tracks are simply

$$E\{z_i z_k^T\} = \mathbf{P} + \mathbf{R}_i \delta_{i,k} \qquad \text{for } i, k = 1, \ldots, N. \quad (10)$$

To represent the likelihood function for this case, we need to introduce some additional notation. Let $Z_{k:l}$ represent the collection of track estimates z_i for $i = k, \ldots, l$, i.e.,

$$Z_{k:l} = [z_k^T, z_{k+1}^T, \ldots, z_{l-1}^T, z_l^T]^T.$$

Also, denote

$$\mathbf{R}_{k:l,m:n} = E\{Z_{k:l} Z_{m:n}^T\}. \quad (11)$$

Now, the likelihood function is given by

$$p(z_1, \ldots, z_N | H_1) = p(z_1 | H_1) p(z_2 | Z_{1:1}, H_1) \cdots \cdots p(z_i | Z_{1:i-1}, H_1) \cdots p(z_N | Z_{1:N-1}, H_1), \quad (12)$$

where

$$p(z_i | Z_{1:i-1}, H_1) = \frac{1}{(2\pi)^{\frac{M}{2}} |\mathbf{C}_i|^{\frac{1}{2}}} \exp\left(-\frac{1}{2} \cdot \times (z_i - E\{z_i | Z_{1:i-1}\})^T \mathbf{C}_i^{-1} (z_i - E\{z_i | Z_{1:i-1}\}) \right), \quad (13)$$

and

$$\mathbf{C}_i = \mathbf{R}_{i:i,i:i} - \mathbf{R}_{i:i,1:i-1}\mathbf{R}^{-1}_{1:i-1,1:i-1}\mathbf{R}^T_{i:i,1:i-1}.$$

The conditional expectations are given by

$$E\{z_i|Z_{1:i-1}\} = \mathbf{R}_{i:i,1:i-1}\mathbf{R}^{-1}_{1:i-1,1:i-1}Z_{1:i-1}.$$

Using (10) and (11), it is clear that

$$\mathbf{R}_{i:i,i:i} = \mathbf{R}_i + \mathbf{P},$$

$$\mathbf{R}_{i:i,1:i-1} = [\mathbf{P}\ldots\mathbf{P}]^T,$$

and

$$\mathbf{R}_{1:i-1,1:i-1} = \begin{bmatrix} \mathbf{R}_1+\mathbf{P} & \mathbf{P} & \ldots & \mathbf{P} \\ \mathbf{P} & \mathbf{R}_2+\mathbf{P} & \ldots & \mathbf{P} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{P} & \mathbf{P} & \ldots & \mathbf{R}_{i-1}+\mathbf{P} \end{bmatrix}.$$

As a result, it can easily be verified that the conditional expectation can be expressed as

$$E\{z_i|Z_{1:i-1}\} = \left(\mathbf{P}^{-1} + \sum_{j=1}^{i-1}\mathbf{R}_j^{-1}\right)^{-1}\sum_{j=1}^{i-1}\mathbf{R}_j^{-1}z_j, \tag{14}$$

and the covariance $\mathbf{C}_i$ can be expressed as

$$\begin{aligned}\mathbf{C}_i &= \mathbf{R}_i + \mathbf{P} - \left(\mathbf{P}^{-1} + \sum_{j=1}^{i-1}\mathbf{R}_j^{-1}\right)^{-1}\left(\sum_{j=1}^{i-1}\mathbf{R}_j^{-1}\right)\mathbf{P} \\ &= \mathbf{R}_i + \mathbf{P} - \mathbf{P}\left(\left(\sum_{j=1}^{i-1}\mathbf{R}_j^{-1}\right)^{-1} + \mathbf{P}\right)^{-1}\mathbf{P}\end{aligned}$$

By the matrix inversion lemma [8],

$$\mathbf{C}_i = \mathbf{R}_i + \left(\mathbf{P}^{-1} + \sum_{j=1}^{i-1}\mathbf{R}_j^{-1}\right)^{-1}.$$

When using the prior statistics given by (9) and letting the variance go to infinity, the likelihood goes to zero. However, the rate of convergence to zero is independent of the track estimates. When using the likelihood as the cost function for the assignment problem, the likelihood is scaled by a constant to get

$$J_\sigma(Z_{1:N}) = \sigma^M p(Z_{1:N}|H_1)$$

as the cost function. Then, as σ^2 goes to infinity, the cost function J_σ approaches J, where

$$\begin{aligned}J(Z_{1:N}) = &\frac{1}{(2\pi)^{\frac{MN}{2}}}\prod_{i=2}^{N}\frac{1}{|\mathbf{R}_i + \mathbf{R}_{f,1:i-1}|^{\frac{1}{2}}}\exp\Big(-\frac{1}{2} \\ &\times (z_i - z_{f,i-1})^T(\mathbf{R}_i + \mathbf{R}_{f,1:i-1})^{-1}(z_i - z_{f,i-1})\Big),\end{aligned}$$

$z_{f,1:i-1}$ is the fused track estimate (see (4) and (6)), and $\mathbf{R}_{f,1:i-1}$ is the fused track covariance (see (5)).

The LRT becomes equivalent to finding the N tracks that minimizes

$$\begin{aligned}c(\mathcal{Z}_1,\ldots,\mathcal{Z}_N) = &\sum_{i=2}^{N}\big[\log(|\mathbf{R}_i + \mathbf{R}_{f,1:i-1}|) \\ &+ (z_i - z_{f,i-1})^T(\mathbf{R}_i + \mathbf{R}_{f,1:i-1})^{-1}(z_i - z_{f,i-1})\big].\end{aligned} \tag{15}$$

Using the matrix inversion lemma, equivalent forms of the cost function can be derived. Specifically, the inverse of $\mathbf{R}_i + \mathbf{R}_{f,1:i-1}$ has the following equivalent representations:

$$\begin{aligned}(\mathbf{R}_i + \mathbf{R}_{f,1:i-1})^{-1} &= \mathbf{R}_i^{-1} - \mathbf{R}_i^{-1}\mathbf{R}_{f,1:i}\mathbf{R}_i^{-1}, \\ &= \mathbf{R}_{f,1:i-1}^{-1} - \mathbf{R}_{f,1:i-1}^{-1}\mathbf{R}_{f,1:i}\mathbf{R}_{f,1:i-1}^{-1}, \\ &= \mathbf{R}_i^{-1}\mathbf{R}_{f,1:i}\mathbf{R}_{f,1:i-1}^{-1}, \\ &= \mathbf{R}_{f,1:i-1}^{-1}\mathbf{R}_{f,1:i}\mathbf{R}_i^{-1}.\end{aligned}$$

Then, (15) can be rewritten as

$$\begin{aligned}c(\mathcal{Z}_1,\ldots,\mathcal{Z}_N) = \sum_{i=2}^{N}\Big[&\log(|\mathbf{R}_i|) + \log(|\mathbf{R}_{f,1:i-1}|) \\ &- \log(|\mathbf{R}_{f,1:i}|) + z_i^T\mathbf{R}_i^{-1}z_i \\ &+ z_{f,1:i-1}^T\mathbf{R}_{f,1:i-1}^{-1}z_{f,1:i-1} \\ &- z_{f,1:i}^T\mathbf{R}_{f,1:i}^{-1}z_{f,1:i}\Big]\end{aligned}$$

Simplifying the sum of telescoping series, the cost function becomes

$$\begin{aligned}c(\mathcal{Z}_1,\ldots,\mathcal{Z}_N) = &-\log(|\mathbf{R}_{f,1:N}|) + \sum_{i=1}^{N}\Big[\log(|\mathbf{R}_i|) \\ &+ z_i^T\mathbf{R}_i^{-1}z_i\Big] - z_{f,1:N}^T\mathbf{R}_{f,1:N}^{-1}z_{f,1:N}.\end{aligned} \tag{16a}$$

or, equivalently,

$$\begin{aligned}c(\mathcal{Z}_1,\ldots,\mathcal{Z}_N) = &-\log(|\mathbf{R}_{f,1:N}|) + \sum_{i=1}^{N}\Big[\log(|\mathbf{R}_i|) \\ &+ (z_i - z_{f,1:N})^T\mathbf{R}_i^{-1}(z_i - z_{f,1:N})\Big].\end{aligned} \tag{16b}$$

Interestingly, the difference between the cost function derived by the generalized likelihood (see (7)) and the one derived by the limit of the likelihood is the extra term in (16) representing the log determinant of the fused covariance for the tracks under test.

The sequential assignment method described in [7] is a suboptimal method that attempts to minimize (16). This is clear by inspecting the equivalent form of the cost function given by (15). The sequential assignment employs the 2-D assignment to minimize each term in (15), where the minimization is implemented sequentially starting with $i = 2$. Unlike the implementation of the S-D assignment to minimize (15), the sequential approach is suboptimal.

4 Simulations

In general, the four cost functions for N track association given by (2), (3), (7) and (16) are different. For N=2, (2), (3) and (16) are identical to (1). In addition, for the special case that all tracks have equal covariance errors, then it can be shown that (3) and (16) lead to equivalent tests. This section uses simulations to illustrate the advantages of (16) in more general cases.

The first simulation considers the track assignment application [1]. An assignment algorithm groups tracks from different sensors so that the sum of the cost functions associated to each group is minimized. In this simulation, a simple illuminating case with the probability of detecting a true track of one (*i.e.,* $P_d = 1$) and the probability of detecting a false track of zero (*i.e.,* $P_{fa} = 0$) is considered. In addition, the error covariance for all tracks from the same sensor are equal. Finally, the number of sensors is three and the number of tracks is two. This assignment problem is a maximum likelihood test (or minimum cost) of four possible track association hypotheses as illustrated in Table 1.

For this scenario, (7) and (16) are equivalent, but different from (2) and (3). The equivalency occurs because the log determinant terms in the cost function are constant with respect to the different assignment possibilities. Therefore, we compare the performance of the assignment algorithm when using either of the likelihood based cost functions ((7) or (16)) to the assignment algorithm when employing the sequential cost of (2) or SAPC of (3).

For this assignment application, two cases for the error covariance are considered: (1) isotropic and (2) anisotropic. Furthermore, the dimension of the target state is $M = 2$, which could represent the two dimensional location of a target on a plane. For the isotropic case, the track error is the same for both dimensions so that the covariance error for the three sensors are $\mathbf{R}_i = \sigma_i^2 \mathbf{I}$ for $i = 1, 2, 3$, where $\sigma_1 = 1$, $\sigma_2 = 2$ and $\sigma_3 = 5$. In a sense, the accuracy of the sensors are different, and the sensors are sorted in order of high, medium and low accuracy, respectively. For the anisotropic case,

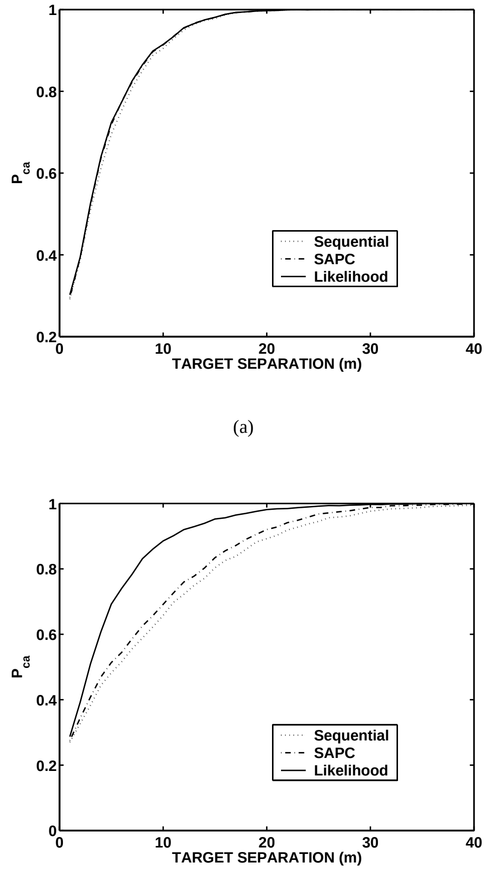
$$\mathbf{R}_i = \mathbf{A}^T(\theta_i) \begin{bmatrix} 100 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{A}(\theta_i),$$

where

$$\mathbf{A}(\theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

is a unitary rotation matrix, $\theta_i = \frac{\pi}{3}(i-1)$, and $i = 1, 2, 3$. This anisotropic case is typical for many radar applications.

For both cases, the number of different possible assignments is four. It can be easily verified that the cost functions are invariant to translations of the target state. Because the number of targets is two, the performance of the assignment algorithms changes only as function of the difference between the two target states. Given that $M = 2$, the magnitude and the orientation of the target state can be different. For a given target separation, 10,000 independent Monte Carlo simulations in which the orientation of the two targets were drawn from a uniform distribution over $[0, 2\pi)$ were run. The resulting probabilities of correct assignment P_{ca} as function of target separations are plotted



(a)

(b)

Fig. 1: Probability of correct assignment versus target separation when using the sequential pairwise sum, SAPC and likelihood cost functions for the case of three sensors and two targets: (a) Isotropic measurement errors and (b) anisotropic measurement errors.

in Fig. 1 for both the isotropic and anisotropic cases when using either the likelihood-based, sequential, or SAPC cost functions. As expected P_{ca} goes to one as the target separations becomes larger. For very small target separation, the assignment algorithms degenerate to random guessing, which results in $P_{ca} = 0.25$ for this four hypotheses assignment problem. For the isotropic case, both cost functions lead to virtually the same performance. However, for the anisotropic case, the likelihood-based cost functions provide a significantly higher P_{ca} for a given target separation. In addition, the SAPC always outperforms the sequential sum of pairwise costs.

The cost function derived from the limit likelihood as the support $\mathcal{S}$ goes to infinity does not necessarily minimize Bayes error when the true prior for the target state is uniform over a bounded region. As discussed in Section 3.5, a closed form expression for the likelihood does not in general exist. If all the error covariances for each measurement of each sensor are equal to the same scaled identity matrix (*i.e.,* $\mathbf{R}_i = \sigma^2 \mathbf{I}$ for $i = 1, 2, 3$) then the optimal cost function when the target state is uniformly distributed over an

Table 1: All possible track assignment hypotheses for tracks $\mathcal{Z}_{j,i}$ where j is the track index for the i-th sensor. Note that $P_d = 1$ and $P_{fa} = 0$.

Hypothesis	Associations		Cost
1	$\mathcal{Z}_{1,1} \sim \mathcal{Z}_{1,2} \sim \mathcal{Z}_{1,3}$	$\mathcal{Z}_{2,1} \sim \mathcal{Z}_{2,2} \sim \mathcal{Z}_{2,3}$	$c(\mathcal{Z}_{1,1}, \mathcal{Z}_{1,2}, \mathcal{Z}_{1,3}) + c(\mathcal{Z}_{2,1}, \mathcal{Z}_{2,2}, \mathcal{Z}_{2,3})$
2	$\mathcal{Z}_{1,1} \sim \mathcal{Z}_{1,2} \sim \mathcal{Z}_{2,3}$	$\mathcal{Z}_{2,1} \sim \mathcal{Z}_{2,2} \sim \mathcal{Z}_{1,3}$	$c(\mathcal{Z}_{1,1}, \mathcal{Z}_{1,2}, \mathcal{Z}_{2,3}) + c(\mathcal{Z}_{2,1}, \mathcal{Z}_{2,2}, \mathcal{Z}_{1,3})$
3	$\mathcal{Z}_{1,1} \sim \mathcal{Z}_{2,2} \sim \mathcal{Z}_{1,3}$	$\mathcal{Z}_{2,1} \sim \mathcal{Z}_{1,2} \sim \mathcal{Z}_{2,3}$	$c(\mathcal{Z}_{1,1}, \mathcal{Z}_{2,2}, \mathcal{Z}_{1,3}) + c(\mathcal{Z}_{2,1}, \mathcal{Z}_{1,2}, \mathcal{Z}_{2,3})$
4	$\mathcal{Z}_{1,1} \sim \mathcal{Z}_{2,2} \sim \mathcal{Z}_{2,3}$	$\mathcal{Z}_{2,1} \sim \mathcal{Z}_{1,2} \sim \mathcal{Z}_{1,3}$	$c(\mathcal{Z}_{1,1}, \mathcal{Z}_{2,2}, \mathcal{Z}_{2,3}) + c(\mathcal{Z}_{2,1}, \mathcal{Z}_{1,2}, \mathcal{Z}_{1,3})$

$a \times a$ region is

$$\begin{aligned} c(\mathcal{Z}_1, \ldots, \mathcal{Z}_3) = & -2\log\Big(\operatorname{erf}\big((a/2 - [z_{f,1:3}]_1)\sqrt{3/2}\big) \\ & + \operatorname{erf}\big((a/2 + [z_{f,1:3}]_1)\sqrt{3/2}\big)\Big) \\ & - 2\log\Big(\operatorname{erf}\big((a/2 - [z_{f,1:3}]_2)\sqrt{3/2}\big) \\ & + \operatorname{erf}\big((a/2 + [z_{f,1:3}]_2)\sqrt{3/2}\big)\Big) \\ & + \sum_{i=1}^{N} (z_i - z_{f,1:3})^T (z_i - z_{f,1:3}), \end{aligned} \quad (17)$$

where $[s]_i$ represents the i-th element of the vector s and $\operatorname{erf}(\cdot)$ is the error function,

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-x^2) dx.$$

Because all the covariances are equal, the three cost functions considered in this paper are equivalent. In addition, as the size of the FOV a goes to infinity, the optimal cost function, as expected, transforms into (16), which is also equivalent to the other cost functions. To illustrate the performance of the cost functions, the P_{ca} of (16) is compared to (17) through Monte Carlo simulations. For a FOV size a, two random target states and the associated tracks for each sensor were sampled and the assignment algorithm is executed. This experiment was repeated 10,000 times for a fixed value of a. Fig. 2 shows that performance of the two cost functions are nearly identical. For most values of a, (17) achieved a slightly higher value of P_{ca}. However, the differences in performance is beyond the statistical significance of the simulation.

For the previous assignment tests, the performance of (7) and (16) are equivalent because the track covariance error is constant for each sensor. Even for this case, the two cost functions do differ when possible false tracks appear or not all of the tracks are detected at each sensor. Lets consider another scenario where two targets exist and two sensors are tracking one target and the other sensor is tracking the second target. The assignment algorithm will need to determine which two sensors are tracking the same target. This is achieved by using (7) or (16) for the case that $N = 2$ and finding the minimum value for all three possibilities of fusing two tracks from three sensors. For the isotropic cases, the two cost functions lead to two different algorithms. For the anisotropic case, the determinants of the covariance matrices are all equal and the two cost functions are equivalent.

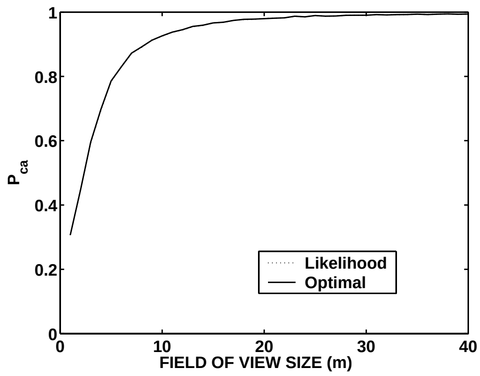


Fig. 2: Probability of correct assignment versus FOV size when using the likelihood cost function (based on no prior knowledge of the target state) and the optimal cost function given knowledge of the sensor's FOV.

Table 2: Confusion matrix for 10,000 Monte Carlo simulations of track assignment when using the GLRT based cost function given by (7).

Estimated / Truth	$1 \sim 2 \mid 3$	$1 \sim 3 \mid 2$	$2 \sim 3 \mid 1$
$1 \sim 2 \mid 3$	3218	205	11
$1 \sim 3 \mid 2$	1448	1689	66
$2 \sim 3 \mid 1$	1644	1432	287

For a given separation distance between the two targets, 10,000 Monte Carlo simulations were ran for the isotropic case. Tables 2 and 3 provides the confusion matrices for the resulting assignments when the target separation is 5 m. In the table, the notation $x \sim y \mid z$ indicates that sensors x and y are tracking the first target and sensor z is tracking the other target. Overall, the probability of correct assignment is 52% and 58% for (7) and (16), respectively. The two tables show that the GLRT biases the decision towards fusing the two most accurate sensors. As a result, the GLRT is able to achieve better performance for the case that the two most accurate sensors are tracking the same target. In contrast, the LRT achieves higher P_{ca} when the two least accurate sensors are actually tracking the same target. Given that all three track association possibilities are equally likely, the average performance of the LRT is better than the GLRT. Fig. 3 confirms this trend through a range of target separations. Overall, (16) leads to slightly better performance than (7).

Table 3: Confusion matrix for 10,000 Monte Carlo simulations of track assignment when using the LRT based cost function given by (16).

Estimated / Truth	$1 \sim 2 \mid 3$	$1 \sim 3 \mid 2$	$2 \sim 3 \mid 1$
$1 \sim 2 \mid 3$	3121	193	120
$1 \sim 3 \mid 2$	1270	1413	520
$2 \sim 3 \mid 1$	1287	816	1260

5 Conclusions

This paper derives the IPL cost, a nearly optimal cost function, for an N sensor track assignment algorithm. The IPL cost can be interpreted as an LRT for the case that the target state can take on any value. One can interpret the sequential assignment method proposed by Blackman as a suboptimal method that attempts to optimize the IPL. The IPL cost was compared with other previously proposed cost functions through Monte Carlo simulations of simple assignment scenarios. For many cases, all cost functions led to virtually identical performance. However, the LRT is superior for cases 1) where the covariance measurement errors are not isotropic, 2) where the probability of detection is less than one, and 3) where the probability of false tracks is greater than zero.

Future work is planned to compare the GLRT and LRT methods for cases, where the track covariance error differs over a single sensor. Our simulations also demonstrated that for a very simple case of equal isotropic track covariance errors, all cost functions lead to optimal performance. For more general scenarios, the optimal cost function must be computed numerically. Comparison of the LRT to the optimal cost function for these more general scenarios is planned.

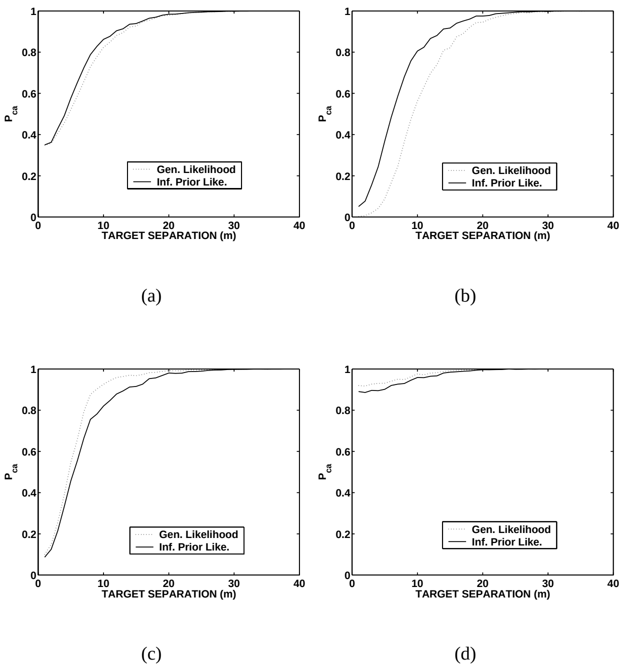


Fig. 3: Probability of correct assignment versus target separation when using the generalized likelihood and likelihood cost functions for the case of three sensors and two targets: (a) Overall results and (b) conditioned on the case that the high and medium error sensors associate ($2 \sim 3 \mid 1$), (c) conditioned on the case that the high and low error sensors associate ($1 \sim 3 \mid 2$) and (d) conditioned on the case that the medium and low error sensors associate ($1 \sim 2 \mid 3$).

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