A Probabilistic Strongest Neighbor Filter Algorithm for *m* Validated Measurements

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Abstract - *The measurement with the strongest signal amplitude in the validation gate is known as the strongest neighbor (SN) measurement. A standard Kalman filter that utilizes the SN at any time as if it is originated from the true target is called the strongest neighbor filter(SNF). Inconsistency of handling the SN as if it is true target is corrected in the existing probabilistic strongest neighbor filter(PSNF) which accounts the probability that the SN is from the true target. It is known that performance of the PSNF is superior to the SNF at a cost of increased computational load. In this paper, we propose a new probabilistic strongest neighbor filter that takes into account the current number of validated measurements in the derivation of probability density functions for the SN which are needed to establish probability weightings and estimation error covariance. The proposed algorithm does not involve infinite summation while the existing PSNF algorithm contains infinite summation that requires approximation for practical usage. Performance of the proposed filter is compared with the existing filters such as the SNF and the PSNF through a series of Monte Carlo simulation runs for aerial target tracking in clutter. The advantages of the new filter in practical applications are studied via analysis and simulation.*

Keywords: SNF, PSNF, PSNF-*m*, data association, target tracking, clutter, performance analysis.

1 Introduction

Target tracking in a clutter environment requires accurate association of target with a measurement for track maintenance. The existing probabilistic strongest neighbor filter (PSNF)[1] accounts for the probability that the strongest neighbor (SN) in the validation gate is originated from the target while the strongest neighbor filter (SNF)[2] assumes at any time that the SN measurement is target-originated. It is known that the existing PSNF is superior to the SNF in performance at a cost of increased computational load. However, the computational complexity is lower than the probabilistic data association filter (PDAF)[3], which accounts for the probability that each measurement in the validation gate is target-originated. This paper proposes a new form of the PSNF, called the PSNF-*m*, which takes into account the probability that the SN measurement selected among the *m* validated measurements in the validation gate is target-originated. The concept is similar to [9] which incorporates the number of validated measurements into design of the probabilistic nearest neighbor filter. The PSNF-*m* considers the current number of measurements in the validation gate whereas the probability in the PSNF is calculated by applying the order statistic and total probability theorem so that it is an averaged value considering all the possible events related to the number of measurements. The proposed PSNF-*m* is shown to have a similar computational load to an approximated or of the PSNF established by truncating the infinite summation term, and they have similar target tracking performance in clutter. Moreover, performance of the PSNF-*m* is less sensitive to the spatial clutter density. This fact provides substantial benefit in practice since the clutter density is either time-variant or space-variant and not known exactly beforehand. The performance of the PSNF-*m* is compared with the PSNF as well as the SNF based on the results of a series of Monte Carlo simulation runs. Sensitivities to the spatial clutter density are tested for the PSNF and the PSNF-*m* by employing mismatched values for the true and guessed densities.

2 The existing PSNF

The SNF assumes that the strongest neighbor (SN) measurement in the validation gate is originated from the target of interest and the SNF utilizes the SN in the update step of a standard Kalman filter (SKF). The SNF is widely used along with the nearest neighbor filter (NNF) [4], due to computational simplicity in spite of its inconsistency of handling the SN as if it is the true target. The PSNF utilizes the SN in the update step however, it accounts for the probability that the SN is target-oriented such that the target state estimate as well as estimation error covariance is updated with a probabilistic weighting factor. It is known that the PSNF is superior in performance to the SNF at a cost of more involved computation.

The validation gate used is the ellipsoid:

$$R_\gamma(k) = \{\nu_k ; \nu_k^T S_k^{-1} \nu_k \le \gamma\} \tag{1}$$

where ν_k is a zero-mean Gaussian residual with covariance of S_k for the true measurement, and $\sqrt{\gamma}$ is called the gate size. The volume of the n-dimensional gate $R_\gamma(k)$ satisfies

$$V_G = C_n \left|S_k\right|^{\frac{1}{2}} \gamma^{\frac{n}{2}} \tag{2}$$

where $C_1 = 2, C_2 = \pi, C_3 = \frac{4}{3}\pi$ etc. The validated measurements consist of n-dimensional location information z and signal amplitude information a. The following assumptions are used in this paper.

A1) The true target signal amplitude is the magnitude-square output of a matched filter so that the signal is χ^2 -distributed with probability density function (pdf)

$$f_1(a) = \frac{1}{1+\rho} e^{-\frac{a}{1+\rho}} \tag{3}$$

where ρ is the signal-to-noise ratio. The clutter signal amplitude satisfies

$$f_0(a) = e^{-a}\,. \tag{4}$$

A2) The number of validated true measurement is denoted by m^T, and m^T is at most 1. The probability that $m^T = 1$ is $P(m^T = 1) = P_D P_G$ where P_D is the probability of target detection indicating that the target signal amplitude exceeds a threshold τ; and P_G is the probability that the target falls inside the validation gate. P_D satisfies $P_D = e^{-\frac{\tau}{1+\rho}}$ from A1), and the probability that the false measurement signal exceeds the threshold τ is $P_{fa} = e^{-\tau}$.

A3) The number of validated false measurements in the validation gate, denoted by m^F, is Poisson distributed with a spatial density λ such that

$$\mu_F(m) = P(m^F = m) = \frac{(\lambda V_G)^m}{m!} e^{-\lambda V_G}\,. \tag{5}$$

A4) The state prediction error $\overline{e}_k = x_k - \overline{x}_k$ for any given time k is a zero-mean Gaussian process with a covariance $\overline{P}_k$ such that $\overline{e}_k \sim N(\overline{e}_k; 0, \overline{P}_k)$.

A5) The validated false measurements at any time are i.i.d. uniformly distributed over the gate.

A6) The location and amplitude of a validated false measurement are independent of the true measurement at any time and other validated false measurements at any other time.

A7) Amplitude is independent of the location.

A8) The target is existing and can be detectable, i.e., it is perceivable [5].

For the SNF and the PSNF, there exist the following three events related to data association with the SN measurement.

M_0: There is no validated measurement;
M_T: The SN measurement is originated from the target;
M_F: The SN measurement is from a false target.

The algorithm of the PSNF derived in [1] is summarized for reference.

<u>The PSNF algorithm</u>

1) Prediction step
identical to the SKF

2) Update step
(a) For the case of M_0

$$\hat{X}_k = \overline{X}_k$$

$$\hat{P}_k = \overline{P}_{k,M0} = \overline{P}_k + \frac{P_D P_G (1 - C_{Tg})}{1 - P_D P_G} K S K^T$$

(b) For the case of $\overline{\mathrm{M}}_0$

$$\hat{X}_k = \overline{X}_k + K \beta_1 \nu$$

$$\hat{P}_k = \overline{P}_k + \left(\frac{P_D P_G P_A (1 - C_{Tg})}{1 - P_D P_G P_A} \beta_0 - \beta_1 \right) K S K^T + \beta_1 \beta_0 K \nu \nu^T K^T$$

$$P_A = \frac{1}{P_D (1 - e^{-\lambda V_G})} (I_A - P_D e^{-\lambda V_G})$$

$$I_A = \int_\tau^\infty e^{-\lambda V_G \left(\frac{e^{-a}}{P_{fa}} \right)} \frac{1}{1+\rho} e^{-\frac{a}{1+\rho}} da = P_D \sum_{j=0}^{\infty} \frac{(-\lambda V_G)^j}{j!(1 + j(1+\rho))}$$

$$\beta_1 = \frac{f(D,a,M_T)}{f(D,a,M_F) + f(D,a,M_T)}\,,\ \beta_0 = 1 - \beta_1$$

$$f(D,a,M_T) = \frac{n V_D}{2D} N(D) e^{-\lambda V_G \left(\frac{e^{-a}}{P_{fa}} \right)} \frac{1}{1+\rho} e^{-\frac{a}{1+\rho}} 1(a - \tau)$$

$$f(D,a,M_F) = \frac{n V_D}{2D} (1 - P_G e^{-\frac{a}{1+\rho}}) \lambda \left(\frac{e^{-a}}{P_{fa}} \right) e^{-\lambda V_G \left(\frac{e^{-a}}{P_{fa}} \right)} 1(a - \tau)$$

In the above, K is the filter gain and C_{Tg} satisfies

$$C_{Tg} = \frac{\int_0^\gamma q^{\frac{n}{2}} e^{-\frac{q}{2}} dq}{n \int_0^\gamma q^{\frac{n}{2}-1} e^{-\frac{q}{2}} dq} \tag{6}$$

hence, $C_{Tg} = (1 - e^{-\frac{\gamma}{2}}(1 + \frac{\gamma}{2}))/(1 - e^{-\frac{\gamma}{2}})$ for $n = 2$. D is the normalized distance squared (NDS) of the SN measurement with location information z^*, so that $D = v^{*T} S^{-1} v^*$, and $V_D = C_n |S|^{\frac{1}{2}} D^{\frac{n}{2}}$ is the volume of the ellipsoid R_D with the gate size $\sqrt{D}$ such as $R_D = \{v; v^T S^{-1} v \le D\}$, $1(x)$ is the unit step function defined as 1 if $x \ge 0$, 0 for elsewhere, and $N(D)$ is the Gaussian pdf of v^* expressed by using the NDS D such that $N(D) = e^{-\frac{D}{2}} / \sqrt{|2\pi S|}$.

3 The PSNF-*m*

One of the drawbacks of the PSNF is in the calculation of I_A the unconditional probability that the SN is target-originated, and it is recommended to use interpolation from a tabulated I_A versus λV_G for various signal-to-noise ratios or to use approximation with the first L terms. In the section a PSNF algorithm which accounts for the number of validated measurements in the update step is derived, and it turns out that the new algorithm is computationally cheaper than the exact from of the existing PSNF with a similar performance in a clutter environment.

Under the conditions that the number of validated measurements is m and the SN measurement is target-oriented, the conditional pdf (cpdf) of the signal amplitude a satisfies (conditioning on the sequence of sets of the past measurements is omitted for brevity)

$$f(a \mid M_T, m) = \frac{1}{P(M_T, m)} f(a, M_T, m). \tag{7}$$

Consider the following Theorem.

Theorem 1: With Assumptions A1) ~ A3), $f(a \mid M_T, m)$ is given by

$$f(a \mid M_T, m) = \frac{1}{P(M_T, m)} P_G \left(1 - \frac{e^{-a}}{P_{fa}}\right)^{m-1} f_1(a) \mu_F(m-1) \tag{8}$$

and the joint probability $P(M_T, m)$ can be obtained as

$$P(M_T, m) = P_D P_G \overline{P}_A \mu_F(m-1) \tag{9}$$

$$\overline{P}_A = \frac{1}{P_D} \int_\tau^\infty f_1(a) \left(1 - \frac{e^{-a}}{P_{fa}}\right)^{m-1} da \tag{10}$$

where $\overline{P}_A$ is the probability that the validated target amplitude $a^T = a$ is the strongest among the m validated measurements under the assumptions of $m^T = 1$ and $m^F = m - 1$.

Proof: See Appendix.

Note that for $m = 1$, $\overline{P}_A$ is 1 such that $P(M_T, m) = P(m^T = 1) = P_D P_G$.

For $f(a \mid M_F, m)$, one can state the following Theorem.

Theorem 2: With Assumptions A1) ~ A3), $f(a \mid M_F, m)$ is given by

$$\begin{aligned} f(a \mid M_F, m) &= \frac{1}{P(M_F, m)} f(a, M_F, m) \\ &= \frac{1}{P(M_F, m)} \begin{bmatrix} (1 - P_D P_G) f_{c_l}(a \mid m) \mu_F(m) \\ + P_G (P_D - e^{-\frac{a}{1+\rho}}) f_{c_l}(a \mid m-1) \mu_F(m-1) \end{bmatrix} \end{aligned} \tag{11}$$

where $f_{c_l}(a \mid m)$ denotes the cpdf of amplitude a of the clutter-originated SN measurement under the assumption that the number of validated false measurements $m^F = m$, and $f_{c_l}(a \mid m)$ satisfies

$$f_{c_l}(a \mid m) = m \frac{e^{-a}}{P_{fa}} \left(1 - \frac{e^{-a}}{P_{fa}}\right)^{m-1}, \tag{12}$$

and $P(M_F, m)$ of (11) is expressed as

$$P(M_F, m) = (1 - P_D P_G) \mu_F(m) + P_D P_G (1 - \overline{P}_A) \mu_F(m-1). \tag{13}$$

Proof: See Appendix.

Remark 1: The probability that the number of validated measurements in the validation gate at any time can be obtained by utilizing (9) and (13) as

$$\begin{aligned} P(m) &= P(M_T, m) + P(M_F, m) \\ &= (1 - P_D P_G) \mu_F(m) + P_D P_G \mu_F(m-1) \end{aligned}. \tag{14}$$

The updated error covariance matrices for M_0 and M_T of the PSNF-m are identical to the ones for the PSNF however, the error covariance for M_F is different. It is required to derive the cpdf of D^t, the NDS of the target, under the assumptions of M_F and m validated measurements for the error covariance calculation. Since the target under consideration is perceivable by Assumption A8), the target is not temporarily obscured and it is not proper to use merely the predicted error covariance for M_F.

Under the perceivability assumption, the following events may occur for M_F:

1) The target may be located in the validation gate and detected but the signal amplitude a^T is not the strongest among the m validated measurements.
2) The target may be detected but it may not be in the validation gate.
3) The target may not be detected.

Theorem 3: With Assumptions A1) ~ A4), A6), and A8) the cpdf $f(D^t \mid M_F, m)$ is obtained by

$$f(D^t \mid M_F, m) = \frac{\begin{bmatrix} \frac{nV_{D^t}}{2D^t} N(D^t)((1-P_D 1(\gamma - D^t))\mu_F(m) \\ +P_D(1-\overline{P}_A)1(\gamma - D^t)\mu_F(m-1) \end{bmatrix}}{(1-P_D P_G)\mu_F(m) + P_D P_G (1-\overline{P}_A)\mu_F(m-1)} \tag{15}$$

Proof: See Appendix.

Theorem 4: With Assumption A1) ~ A4), A6) and A8), the updated error covariance for M_F is given by

$$\overline{P}_{k,M_F} = \overline{P}_k - KSK^T + \frac{(1-P_D P_G C_{Tg})\lambda V_G + P_D P_G C_{Tg}(1-\overline{P}_A)m}{(1-P_D P_G)\lambda V_G + P_D P_G (1-\overline{P}_A)m} KSK^T \tag{16}$$

Proof: See Appendix.

Note that in the case of $m=0$, $\overline{P}_{k,M_F}$ becomes $\overline{P}_{k,M_0}$ as expected.

Remark 2: $\overline{P}_A$ can be evaluated from (14) as a function of m, the number of validated measurements, such as

$$\overline{P}_A = 1 + \sum_{i=1}^{m-1} (-1)^i C_i^{m-1} \frac{1}{(i+1)+i\rho} \; . \tag{17}$$

$\overline{P}_A$ does not involve infinite summation as seen in the PSNF algorithm so that the PSNF-m is computationally cheaper without approximation. The probability weighting for M_T is evaluated from the a posteriori probabilities as

$$\beta_1 = P(M_T \mid D, a, m) = \frac{f(D,a,M_T,m)}{f(D,a,M_T,m) + f(D,a,M_F,m)} \tag{18}$$

where by Assumptions A5) and A7)

$$\begin{aligned} f(D,a,M_T,m) &= f(D \mid a, M_T, m) f(a, M_T, m) \\ &= f(D \mid M_T) f(a, M_T, m) \\ f(D,a,M_F,m) &= f(D \mid M_F) f(a, M_F, m) \end{aligned} \tag{19}$$

Note that $f(D \mid M_T) = N(D)/P_G$, $f(D \mid M_F) = 1/V_G$, and $f(a,M_T,m)$ and $f(a,M_F,m)$ are expressed in (8) and (11) respectively. The proposed algorithm of the PSNF-*m* is summarized below.

<u>The PSNF-*m* algorithm</u>

1) Prediction step
identical to the SKF

2) Update step
(a) For the case of M_0

$$\hat{X}_k = \overline{X}_k$$

$$\hat{P}_k = \overline{P}_{k,M_0} = \overline{P}_k + \frac{P_D P_G (1-C_{Tg})}{1-P_D P_G} KSK^T$$

(b) For the case of $\overline{\mathrm{M}}_0$

$$\hat{X}_k = \overline{X}_k + K\beta_1 \nu^*$$

$$\overline{P}_{k,M_F} = \overline{P}_k - KSK^T + \frac{(1-P_D P_G C_{Tg})\lambda V_G + P_D P_G C_{Tg}(1-\overline{P}_A)m}{(1-P_D P_G)\lambda V_G + P_D P_G (1-\overline{P}_A)m} KSK^T$$

$$\hat{P}_k = \overline{P}_{k,M_F}(1-\beta_1) + (\overline{P}_k - KSK^T)\beta_1 + \beta_0 \beta_1 K \nu^* \nu^{*T} K^T$$

$$\overline{P}_A = 1 + \sum_{i=1}^{m-1} (-1)^i C_i^{m-1} \frac{1}{(i+1)+i\rho} \quad (\overline{P}_A = 1 \quad for \quad m=1)$$

$$\beta_1 = \frac{P_D N(D) f_1^\tau(a)\left(1 - \frac{e^{-a}}{P_{fa}}\right)}{\begin{bmatrix} \lambda(1-P_D P_G) f_0^\tau(a)\left(1-\frac{e^{-a}}{P_{fa}}\right) \\ + P_G(P_D - e^{-\frac{a}{1+\rho}})(m-1) f_0^\tau(a) \frac{1}{V_G} \\ + P_D N(D) f_1^\tau(a)\left(1-\frac{e^{-a}}{P_{fa}}\right) \end{bmatrix}}$$

$$\beta_0 = 1 - \beta_1 , \; f_1^\tau(a) = \frac{f_1(a)}{P_D}, \; f_0^\tau(a) = \frac{f_0(a)}{P_{fa}} .$$

4 Simulation results

Monte Carlo simulation results of 2-dimensional aerial target tracking in clutter are presented to demonstrate the performance of the proposed PSNF-*m* by comparison with the SNF and the PSNF. The initial position of the target is (7Km, 4Km) and the target is initially moving in a straight line with a speed of 380m/s with 60° of heading angle from the Y-axis. The target is susceptible to lateral maneuver with an acceleration of A_T during tracking. The Singer model [6] is employed for target acceleration dynamic equation for the filters such as

$$\dot{x} = \begin{bmatrix} O_2 & I_2 & O_2 \\ O_2 & O_2 & I_2 \\ O_2 & O_2 & -\frac{1}{\tau} I_2 \end{bmatrix} x + \begin{bmatrix} O_2 \\ O_2 \\ I_2 \end{bmatrix} \begin{bmatrix} w_X \\ w_Y \end{bmatrix} \tag{20}$$

where x is composed of target position, velocity, and acceleration components, $1/\tau$ is a bandwidth of target acceleration and $\tau = 5\,(\mathrm{sec})$. I_2 and O_2 represent 2 × 2 identity matrix and zero matrix, respectively. The process noise $(w_X, w_Y)^T$ is a zero-mean white Gaussian noise vector with the power spectral density of $1.6 \times 10^{-4} I_2 \;\; m^2/\sec^5$. The location information z corrupted by a measurement noise vector can be described by

$$z_k = (I_2, O_2, O_2)x + \upsilon_k \tag{21}$$

where υ_k is a zero-mean white Gaussian noise vector sequence with covariance of $(20)^2 m^2 I_2$. The sampling frequency for target tracking is chosen to be $10Hz$.

Table 1 is a summary of track loss percentages obtained from 500 runs of Monte Carlo simulation resulting from employing the SNF, the PSNF, and the PSNF-*m* for the cases of fixed $\rho = 10$, $\gamma = 9$, $A_T = 0$ and varying P_D and λ. Track loss is declared if the position estimation error in the X or Y axis exceeds 10 times the standard deviation of the measurement noise. The results indicate that the performance of the SNF gets worse for lower P_D and larger λ while the PSNF and the PSNF-*m* show similar and excellent tracking performance. Simulation results with different parameter sets indicate similar characteristics.

Table 1: Track loss percentage

P_D	λ	SNF	PSNF	PSNF-*m*
0.7	0.00005	10.4	0	0
	0.0001	13.4	0	0
	0.00015	17.4	0	0
	0.0002	22.6	0	0
	0.0003	23.0	0	0
0.8	0.00005	6.4	0	0
	0.0001	9.2	0	0
	0.00015	13.4	0	0
	0.0002	14.8	0	0
	0.0003	16.0	0	0
0.9	0.00005	2.8	0	0
	0.0001	5.4	0	0
	0.00015	9.6	0	0
	0.0002	11.2	0	0
	0.0003	13.4	0	0

Since it is hard to choose the correct λ for a clutter environment in practice, many on-line estimation algorithms for λ have been suggested [7]. However they may cost an increased computational load, and convergence may take too long for environment changes. The results of sensitivity to λ for the PSNF and the PSNF-*m* are summarized in Table 2. The true λ is chosen as λ=0.00015 while each filter uses a guessed value $\hat{\lambda}$. Target undergoes lateral 2g-maneuver at $t = 3\sec$ in this case and the zero-mean process noise vector is modeled with the power spectral density of $2.13 I_2 m^2/\sec^5$ to accommodate target maneuver. Table 2 idicates the track loss percentages obtained from 500 Monte Carlo simulation runs for case I and case II. For case I, P_D =0.7, ρ =10 and various values of $\hat{\lambda}$ are used while P_D =0.7, ρ =5 are used for case II. The results show that the PSNF-*m* is less sensitive to $\hat{\lambda}$, which indicates that the PSNF-*m* has substantial advantages in practical usage.

Table 2: Sensitivity to λ in terms of track loss percentages

$\hat{\lambda}$ $(\times 10^{-4})$		0.5	1.0	1.5	2.0	4.5	6.5	7.5
Case I	PSNF	18.6	17.8	13.4	11.2	13.8	12.4	11.6
	PSNF-*m*	20.6	16.0	14.2	9.0	7.2	5.4	7.4
Case II	PSNF	40.6	29.4	25.6	23.4	22.4	22.2	21.8
	PSNF-*m*	39.4	26.0	22.4	22.8	14.0	10.6	10.8

5 Conclusions

A new PSNF, called the PSNF-*m*, based on the current number of validated measurements in the gate is derived in this paper. Simulation results show the PSNF-*m* has similar performance to the existing PSNF with less computational complexity. It is found that the PSNF-*m* is less sensitive to the spatial clutter density due to the fact that the number of measurements in the current validation gate used in the algorithm takes the number of clutters into account and thus it makes the algorithm more adaptable to the current clutter environment. Therefore, the PSNF-*m* has advantages in practical applications for which the exact density is not known beforehand or it is not fixed in time and space.

Appendix

A. Proof of Theorem 1

Under M_T, the number of validated true measurement m^T should be 1 and $m^F = m-1$. From the Bayes' rule [8], $f(a, M_T, m)$ of (7) becomes

$$\begin{aligned} f(a, M_T, m) = \\ & P(M_T \mid a^T = a, m^T = 1, m^F = m-1) \\ & \times f(a \mid m^T = 1, m^F = m-1) P(m^T = 1) P(m^F = m-1) \end{aligned} \tag{A-1}$$

where the first term of the RHS implies the probability that all the $m-1$ false measurements in the validation gate have signal amplitudes larger than the threshold τ and smaller than $a^T = a$,

$$P(M_T \mid a^T = a, m^T = 1, m^F = m-1) = \left(1 - \frac{e^{-a}}{P_{fa}}\right)^{m-1} \tag{A-2}$$

where the probability that a validated false measurement has signal amplitude smaller than $a^T = a$ is equal to $1 - \frac{e^{-a}}{P_{fa}}$ is used. $f(a \mid m^T = 1, m^F = m-1)$ in (A-1) is equal to $\frac{1}{P_D} f_1(a)$ and $P(m^T = 1) = P_D P_G$, $P(m^F = m-1) = \mu_F(m-1)$

from Assumptions A2) and A3). Therefore, inserting (A-1) into (7) becomes

$$f(a\,|\,M_T,m) = \frac{1}{P(M_T,m)} P_G \left(1-\frac{e^{-a}}{P_{fa}}\right)^{m-1} f_1(a)\mu_F(m-1) \quad (8)$$

where $P(M_T,m)=\int_\tau^\infty f(a,M_T,m)da$. If we denote $\overline{P}_A$ as the probability that the validated true measurement has the largest signal amplitude among the m validated measurements, $\overline{P}_A$ becomes

$$\overline{P}_A = \int_\tau^\infty f(a^F < a^T = a\,|\,m^T=1, m^F=m-1)da \qquad \text{(A-3)}$$

where a^F is the signal amplitude of any clutter and

$$\begin{aligned} & f(a^F < a^T = a\,|\,m^T=1, m^F=m-1) \\ & = P(a^F \langle a\,|\,m^F=m-1) f(a^T=a\,|\,m^T=1) \\ & = \left(1-\frac{e^{-a}}{P_{fa}}\right)^{m-1} \frac{1}{P_D} f_1(a). \end{aligned} \qquad \text{(A-4)}$$

Inserting (A-4) into (A-3) leads $P(M_T,m)$ in (8) to satisfy

$$P(M_T,m) = P_D P_G \overline{P}_A \mu_F(m-1). \qquad (9)$$

B. Proof of Theorem 2

Under M_F, $m^F \ge 1$ and $m^T=1$ or 0 among the m validated measurements. By the Bayes' rule, $f(a,M_F,m)$ in (11)

$$\begin{aligned} & f(a,M_F,m) \\ & = P(M_F\,|\,a^F=a,m^T=0,m^F=m) f_{c_l}(a\,|\,m^F=m) \\ & \quad \times P(m^T=0)\mu_F(m) \\ & + P(M_F\,|\,a^F=a,m^T=1,m^F=m-1) f_{c_l}(a\,|\,m^F=m-1) \\ & \quad \times P(m^T=1)\mu_F(m-1) \end{aligned}$$

(A-5)

where $f_{c_l}(a\,|\,m^F=m)$ represents the cpdf of a, the signal amplitude of SN measurement associated with a clutter conditioned on m, the number of validated measurements.

$f_{c_l}(a\,|\,m^F=m)$ is given by,

$$f_{c_l}(a\,|\,m^F=m) = \frac{m!}{1!(m-1)!} \frac{f_0(a)}{P_{fa}} \left(1-\frac{e^{-a}}{P_{fa}}\right)^{m-1} \qquad (12)$$

Similarly, $f_{c_l}(a\,|\,m^F=m)$ is obtained by replacing m to $m-1$ from (12).

Note that $P(M_F\,|\,a^F=a,m^T=0,m^F=m)=1$, $P(m^T=0)=1-P_D P_G$, $P(m^T=1)=P_D P_G$.

$P(M_F\,|\,a^F=a,m^T=1,m^F=m-1)$ in the second term of the RHS of (A-5) is rewritten as

$$\begin{aligned} & P(M_F\,|\,a^F=a,m^T=1,m^F=m-1) = P(a^T<a\,|\,m^T=1) \\ & = \frac{1}{P_D}\int_\tau^a f_1(a)da \\ & = 1-\frac{1}{P_D} e^{-\frac{a}{1+\rho}} \end{aligned}$$

(A-6)

Therefore, from (A-6), (12) and (A-5), $f(a\,|\,M_F,m)$ satisfies (11) and $P(M_F,m)$ is obtained from $f(a,M_F,m)$ of (A-5) by

$$P(M_F,m) = \int_\tau^\infty f(a,M_F,m)da \qquad \text{(A-7)}$$

which results in (13).

C. Proof of Theorem 3

Under the perceivability assumption A8), the cpdf of D^t, the NDS of target, conditional on M and m becomes (D^t is denoted as D for brevity)

$$\begin{aligned} f(D\,|\,M_F,m) &= \frac{1}{P(M_F,m)} \frac{nV_D}{2D} N(D) \\ & \times \begin{pmatrix} P_D 1(D-\gamma)\mu_F(m) + P_D(1-\overline{P}_A)1(\gamma-D)\mu_F(m-1) \\ +(1-P_D)\mu_F(m) \end{pmatrix} \end{aligned}$$

(A-8)

where the three terms of the RHS represent the event 2), the event 1), and the event 3) of Section Ⅲ, respectively.

Note that $\frac{nV_D}{2D}$ is the Jacobian used to express the pdf of the NDS D associated with the target. By using $1(D-\gamma)=1-1(\gamma-D)$, and inserting $P(M_F,m)$ of (13) to (A-8), (15) is obtained.

D. Proof of Theorem 4

The cpdf of $\overline{e}_k$, $f(\overline{e}_k\,|\,M_F,m)$ for the corresponding covariance $\overline{P}_{k,M_F}$ can be expressed by using the residual ν^t of the true target measurement (denoted here as ν for brevity)

$$f(\overline{e}_k\,|\,M_F,m) = \int_{\Omega_V} f(\overline{e}_k\,|\,\nu,M_F,m) f(\nu\,|\,M_F,m)d\nu \quad \text{(A-9)}$$

Therefore, $\overline{P}_{k,M_F}$ satisfies,

$$\begin{aligned}\overline{P}_{k,M_F} &= E\left[\overline{e}_k\overline{e}_k^T \mid M_F, m\right] \\ &= \int_{\Omega_{\overline{e}_k}} \overline{e}_k\overline{e}_k^T f(\overline{e}_k \mid M_F, m)d\overline{e}_k \qquad \text{(A-10)} \\ &= \int_{\Omega_\nu}\int_{\Omega_{\overline{e}_k}} \overline{e}_k\overline{e}_k^T f(\overline{e}_k \mid \nu, M_F, m) f(\nu \mid M_F, m) d\overline{e}_k d\nu\end{aligned}$$

Note that $f(\overline{e}_k \mid \nu, M_F, m) = N(\overline{e}_k; K\nu, \overline{P}_k - KSK^T)$ [2] and the cpdf uses ν only regardless of the underlying event M_F and m. Hence

$$\int_{\Omega_{\overline{e}}} \overline{e}_k\overline{e}_k^T f(\overline{e}_k \mid \nu, M_F, m)d\overline{e}_k = \overline{P}_k - KSK^T + K\nu\nu^T K^T . \qquad \text{(A-11)}$$

By inserting (A-11) to (A-10), $\overline{P}_{k,M_F}$ becomes,

$$\overline{P}_{k,M_F} = \int_{\Omega_\nu} (\overline{P}_k - KSK^T + K\nu\nu^T K^T) f(\nu \mid M_F, m) d\nu \qquad \text{(A-12)}$$

where $f(\nu \mid M_F, m)$ can be obtained from $f(D^t \mid M_F, m)$ of (15) by change of variables. (A-12) id further proceeded to

$$\overline{P}_{k,M_F} = \overline{P}_k - KSK^T + \int_{\Omega_\nu} K\nu\nu^T K^T f(\nu \mid M_F, m) d\nu \qquad \text{(A-13)}$$

since integrating $f(\nu \mid M_F, m)$ over Ω_ν results in 1, the ν-independent matrix $\overline{P}_k - KSK^T$ of (A-12) can be placed outside the integral. Now, we would like to apply the same method as [2] to evaluate the integral term of (A-13). Prerequisite equations are summarized as

$$\begin{aligned}&\int_{V_G} K\nu\nu^T K^T N(\nu; 0, S) d\nu \\ &= \frac{1}{n}\int_{V_G} \nu^T S^{-1}\nu \quad N(\nu; 0, S)d\nu \quad KSK^T \qquad \text{(A-14)} \\ &= \frac{1}{n}\left[\frac{nC_n}{2^{\frac{n}{2}+1}\pi^{\frac{n}{2}}}\int_0^\gamma q^{\frac{n}{2}} e^{-\frac{q}{2}} dq\right] KSK^T,\end{aligned}$$

$$\int_{\Omega_\nu} K\nu\nu^T K^T N(\nu; 0, S) d\nu = KSK^T , \qquad \text{(A-15)}$$

$$P_G = \int_{V_G} N(\nu; 0, S)d\nu = \frac{nC_n}{2^{\frac{n}{2}+1}\pi^{\frac{n}{2}}}\int_0^\gamma q^{\frac{n}{2}-1} e^{-\frac{q}{2}} dq , \qquad \text{(A-16)}$$

and C_{Tg} of (6) satisfies

$$C_{Tg} = \frac{\int_{V_G} \nu^T S^{-1}\nu N(\nu; 0, S) d\nu}{nP_G}. \qquad \text{(A-17)}$$

Note that $\frac{nV_{D^t}}{2D^t}N(D^t)$ is the pdf of the NDS $D^t = \nu^T S^{-1}\nu$ where ν is a Gaussian process such that $\nu \sim N(\nu; 0, S)$. $\frac{nV_{D^t}}{2D^t}N(D^t)1(\gamma - D^t)$ is equivalent to $N(\nu; 0, S)$ defined inside the validation gate only.

From (15), $f(\nu \mid M_F, m)$ can be obtained as

$$\begin{aligned}&f(\nu \mid M_F, m) = \frac{1}{P(M_F, m)} N(\nu; 0, S) \\ &\times \left((1 - P_D 1(\nu; R_\gamma))\mu_F(m) + P_D(1 - \overline{P}_A)1(\nu; R_\gamma)\mu_F(m-1)\right)\end{aligned} \qquad \text{(A-18)}$$

where $1(\nu; R_\gamma)$ is a multivariate unit step function defined as 1 for $\nu \in R_\gamma$, 0 for elsewhere. Applying (A-18) to the last term of (A-13) and utilizing (A-14) ~ (A-17), one can obtain

$$\begin{aligned}&\overline{P}_{k,M_F} = \overline{P}_k - KSK^T \\ &+ \frac{(1 - P_D P_G C_{Tg})\mu_F(m) + (1 - P_D P_G C_{Tg}(1 - \overline{P}_A)\mu_F(m-1))}{(1 - P_D P_G)\mu_F(m) + (1 - P_D P_G(1 - \overline{P}_A)\mu_F(m-1))} KSK^T\end{aligned} \qquad \text{(A-19)}$$

which is further reduced to (16) by using $\mu_F(m)$ of (5).

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