

Decision Fusion Rules in Multi-hop Wireless Sensor Networks

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Abstract – We consider in this paper the decision fusion problem for a wireless sensor network (WSN) operating in a fading environment. In particular, we develop channel-aware decision fusion rules for a resource constrained WSN where decisions from local sensors may go through multi-hop transmission to reach a fusion center. Each relay node employs a binary relay scheme whereby the relay output is inferred from the channel impaired observation received from its source node. This estimated binary decision is subsequently transmitted to the next node until it reaches the fusion center. Under a flat fading channel model, we derive the optimum fusion rules at the fusion center for the following two cases. In the first case, we assume that the fusion center has knowledge of the fading channel gains at all hops; while in the second case, assuming a Rayleigh fading model, we derive fusion rules utilizing only the fading channel statistics. We show that both optimum decision fusion statistics reduce to respective simple nonlinearities in the low channel SNR regime that are easy to implement. Performance evaluation, including a study of the robustness of the fusion statistics with respect to unknown system parameters, is conducted through simulations.

Keywords: Wireless sensor networks, decision fusion, data fusion, fading channels, multi-hop transmission.

1 Introduction

Wireless sensor networks have generated an enormous interest from researchers in various disciplines in recent years. Distributed sensors with wireless connection allow joint processing of temporally and spatially distributed information, thus greatly expanding our ability to sense and understand the environment and other complex systems. This promise has spurred intensive research in various areas related to WSN. Current and future applications range from battlefield surveillance, health care and telemedicine, to environmental and habitat monitoring and control [1–5].

A distinct feature of WSN is that wireless communication networks become an integral component of the WSN. This is especially true for resource constrained WSN where a divide and conquer approach (i.e., treating communication network as an independent entity) may lead to significantly inferior performance as well as potential waste of limited resources. The integration of information transmission and processing appears to be a promising direction for

optimized system performance under given resource constraints. This has been investigated both at the fusion center level [6, 7] and at the sensor level [8]. Specifically, in [6, 7], channel aware decision fusion rules have been developed using a canonical distributed detection system where binary decisions from multiple parallel sensors are transmitted through fading channels to a fusion center where they are then combined for final decision making. The canonical parallel fusion structure, while theoretically important and analytically tractable, may not reflect the way a real WSN operates. In most WSN applications, resource constraints, especially the energy constraint in applications involving in-situ unattended sensors operating on irreplaceable power supplies, often dictate that transmission range for each sensor node be limited. Therefore, in order to reach a fusion node, a decision made at a local sensing node may need to go through multiple hops for minimal energy consumption. This is true because radio transmission is one of the major power consumers among all the functions for a sensor node, and the fact that the required transmission power is not linear in the distance between the transmitter and the receiver. The objective of this work is, therefore, to extend the channel aware decision fusion rules developed in [6, 7, 9] to more realistic WSN models that involve multi-hop transmissions.

For multi-hop transmissions, the relay nodes (intermediate nodes between the sensing node and the fusion node) are to convey the information received from its source node to its destination node. Under an ideal situation, each relay node recovers the original decision correctly and hence the fact that local decisions undergo multiple hops does not have any impact on the fusion center design. However, the assumption of reliable relaying is overly optimistic in light of the limited resources and stringent delay requirement, as well as the potential severity of channel impairments. Thus, in this work, we assume a simple memoryless relay scheme where each relay node decides what to transmit using its own observation by employing a maximum likelihood estimate. This estimated decision may be inconsistent with what was originally transmitted and this has to be taken into account in the fusion rule design. Assuming the above binary relay scheme and a flat fading channel model, in this paper we derive the optimal decision fusion algorithms for the following two cases. In the first case, we assume that the fading channel gains for all the hops are available at the

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fusion center and derive the optimal likelihood ratio (LR) based fusion statistic. In the second case, we relax the requirement to knowing only the fading channel statistics by assuming a Rayleigh fading channel model and derive the corresponding optimal LR fusion rule. We emphasize here that the flat fading channel assumption is valid for many WSN applications where sensors are densely deployed in an open field that has a very small delay spread.

The LR-based optimal fusion rules obtained for both cases still require a significant amount of prior information that is either not available or, in some cases, can only be acquired at a cost level that is not permissible for real WSN systems. As such, by imposing additional assumptions, we further reduce the LR-based fusion rules to some simple nonlinearities in a low SNR regime. The organization of the paper is as follows. In the next section, we review the previous work on fusion rule design for a canonical parallel distributed detection system with single hop transmission between sensor nodes and the fusion center. In Sec. 3, we lay out the model for a multi-hop based sensor fusion network. The case of known fading channel amplitude is treated in Sec. 4, followed by the case where only the fading channel statistics are known in Sec. 5. Performance evaluation is given in Sec. 6 and we conclude in Sec. 7.

2 Review of previous work

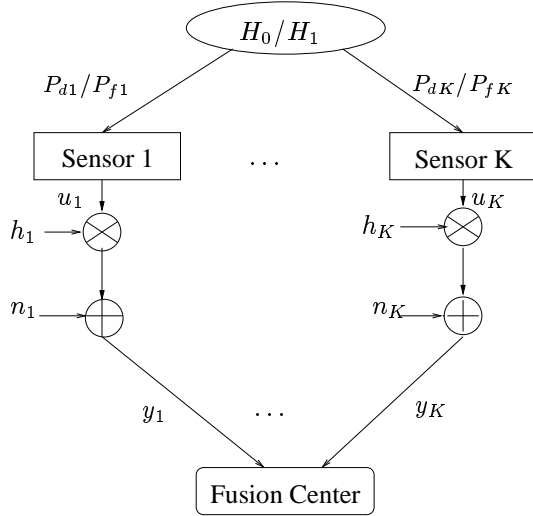


Fig. 1: The canonical single-hop fusion model in the presence of fading channels.

Fig. 1 depicts a typical parallel fusion structure where a flat fading channel model is assumed between each sensor and the fusion center. Assume that the k^{th} local sensor makes a binary decision $u_k \in \{+1, -1\}$, with false alarm and detection probabilities P_{f_k} and P_{d_k} , respectively. That is, we have $P[u_k = 1|\mathbf{H}_0] = P_{f_k}$ and $P[u_k = 1|\mathbf{H}_1] = P_{d_k}$. The received signal at the fusion center from the k^{th} sensor is

$$y_k = h_k u_k + n_k \quad (1)$$

where h_k is the channel fading envelope and n_k is zero-mean additive Gaussian noise with variance σ^2 . Using

the above fusion model, we can obtain the following set of five decision fusion rules, depending on the amount of prior knowledge available [6, 7]. Throughout this paper, we use $\Lambda^{(s)}$ to denote the fusion statistics for the single hop transmission model in order to distinguish from that for the multi-hop systems.

1. Optimal LR-based fusion statistic using complete prior knowledge

Assuming complete channel knowledge, the optimal LR-based fusion statistic was derived in [6, 9]

$$\Lambda_1^{(s)} = \prod_{k=1}^K \frac{P_{d_k} e^{-\frac{(y_k - h_k)^2}{2\sigma^2}} + (1 - P_{d_k}) e^{-\frac{(y_k + h_k)^2}{2\sigma^2}}}{P_{f_k} e^{-\frac{(y_k - h_k)^2}{2\sigma^2}} + (1 - P_{f_k}) e^{-\frac{(y_k + h_k)^2}{2\sigma^2}}} \quad (2)$$

where $\mathbf{y} = [y_1, \dots, y_K]^T$ is a vector containing observations received from all K sensors.

2. LR-based fusion rules using only fading statistics for Rayleigh fading channel

Implementing the optimal LR test as in (2) requires that all *a priori* information, including the instantaneous channel gains, is available. Under the Rayleigh fading model, the LR-based fusion statistic using only the fading parameter is summarized below [7].

Theorem 1 *The LR for decision fusion under the Rayleigh fading channel model is in the form of Eq. (3) (at the top of the next page), where $t = \frac{\sigma_c}{\sigma_n \sqrt{\sigma_c^2 + \sigma_n^2}}$*

with $2\sigma_c^2$ being the mean square value of the fading channel, σ_n^2 is the noise variance, and $Q(\cdot)$ is the complementary distribution function of a standard Gaussian random variable.

3. A two-stage approximation using the Chair-Varshney fusion rule

A direct alternative to the above LR-based fusion rules is to consider the information transmission and decision fusion as a two-stage process — first y_k is used to infer about u_k ; then, the estimates of u_k are employed in the optimum fusion rule. Given the model in (1), the maximum likelihood estimate for u_k is $\hat{u}_k = \text{sign}(y_k)$. Applying the fusion rule derived in [10], herein termed the Chair-Varshney fusion rule, we obtain the following statistic

$$\Lambda_3^{(s)} = \sum_{y_k < 0} \log \left(\frac{1 - P_{d_k}}{1 - P_{f_k}} \right) + \sum_{y_k > 0} \log \left(\frac{P_{d_k}}{P_{f_k}} \right) \quad (4)$$

Not surprisingly, $\Lambda_3^{(s)}$ can be shown to be mathematically equivalent to the two LR-based fusion rules in the large SNR regime (i.e., $\sigma^2 \rightarrow 0$) [6, 7].

4. Fusion statistic using a maximum ratio combiner (MRC)

In the low SNR regime ($\sigma^2 \rightarrow \infty$), we can show that $\Lambda_1^{(s)}$ reduces to $\hat{\Lambda}_4^{(s)} = \sum_{k=1}^K (P_{d_k} - P_{f_k}) h_k y_k$. Further, if the local sensors are identical, i.e., P_{d_k} and P_{f_k}

$$\Lambda_2^{(s)} = \prod_{k=1}^K \frac{P_{dk} \left[1 + \sqrt{2\pi} t y_k e^{\frac{t^2 y_k^2}{2}} Q(-y_k t) \right] + (1 - P_{dk}) \left[1 - \sqrt{2\pi} t y_k e^{\frac{t^2 y_k^2}{2}} Q(t y_k) \right]}{P_{fk} \left[1 + \sqrt{2\pi} t y_k e^{\frac{t^2 y_k^2}{2}} Q(-y_k t) \right] + (1 - P_{fk}) \left[1 - \sqrt{2\pi} t y_k e^{\frac{t^2 y_k^2}{2}} Q(t y_k) \right]} \quad (3)$$

are the same for all k 's, then $\Lambda_1^{(s)}$ reduces to a form analogous to a maximum ratio combiner.

$$\Lambda_4^{(s)} = \frac{1}{K} \sum_{k=1}^K h_k y_k \quad (5)$$

5. Fusion statistic using an equal gain combiner (EGC)

At low SNR ($\sigma^2 \rightarrow \infty$), $\Lambda_2^{(s)}$ in (3) reduces to $\hat{\Lambda}_5^{(s)} = \sum_{k=1}^K (P_{dk} - P_{fk}) \sqrt{2\pi} t y_k$. Further, if the local sensors are identical, i.e., P_{dk} and P_{fk} are the same for all k 's, then $\Lambda_2^{(s)}$ further reduces to a form analogous to an equal gain combiner [7].

$$\Lambda_5^{(s)} = \frac{1}{K} \sum_{k=1}^K y_k \quad (6)$$

3 Multi-hop decision fusion problem

Consider a decision fusion network with multi-hop transmissions as illustrated in Fig. 2. Each sensing node observes data generated according to one of the two hypotheses under test, makes a local decision, and transmits the decision to a fusion center through several relay nodes. Each relay node tries to retrieve the decision sent from its source node from fading and noise impaired observation and relays it to the next node until it reaches the fusion center. The following assumptions are used in our analysis:

1. Binary transmission problem: all sensors (including local sensors and their relay nodes) make a binary decision which is either +1 or -1.
2. All the channels follow by Rayleigh flat fading with unit mean square value, i.e., $E[(h_k^i)^2] = 1, k = 1, 2, \dots, K; i = 0, 1, \dots, M_k$;
3. Noise processes on all the channels are Gaussian with zero mean and variance σ^2 , and are independent of each other;
4. Phase coherent reception, i.e., the effect of a fading channel is simplified as a real scalar multiplication given that the transmitted signal is assumed to be binary;
5. Relay nodes do not directly observe the target. Further, if a relay is used by two or more different sensor-to-fusion center paths, it is assumed that the relay processing is done independently for each path.

With the above assumptions, we can formulate our multi-hop decision fusion network model as follows.

Suppose there are M_k relay nodes corresponding to the k^{th} local sensor, then the number of hops from the k^{th} local sensor to the fusion center is $M_k + 1$. Let u_k^0 denote the original binary decision of the k^{th} local sensor, while $u_k^i, i = 1, 2, \dots, M_k$, denote the retrieved decisions corresponding to the i^{th} relay node, where i is the hop index. Let P_{fk} and P_{dk} denote the false alarm and detection probability for the k^{th} local sensor, i.e., $P[u_k^0 = 1|H_0] = P_{fk}$ and $P[u_k^0 = 1|H_1] = P_{dk}$, with $k = 1, 2, \dots, K$, where K is the total number of local sensors.

For each relay node, we assume a simple binary output which is the maximum likelihood estimate of the decision sent from its source node. Hence, given that the noise is Gaussian, we have

$$u_k^i = \text{sign}(u_k^{i-1} h_k^{i-1} + n_k^{i-1})$$

Let y_k denote the input of the fusion center corresponding to the k^{th} local sensor, thus,

$$y_k = u_k^{M_k} h_k^{M_k} + n_k^{M_k}$$

where h_k^i (a non-negative real number) is the corresponding channel envelope and n_k^i is additive Gaussian noise with zero mean and variance σ^2 .

Our goal is to derive fusion rules using $y_k, k = 1, 2, \dots, K$, that provide robust performance and, at the same time, require as little prior information as possible.

4 Fusion rules with known channel envelope

In this section, we proceed with the derivation of the LR assuming a known channel envelope. Here "known channel envelope" refers to the fact that the instantaneous channel envelope $h_k^i, k = 1, 2, \dots, K; i = 0, 1, \dots, M_k$ are all available at the fusion center. In Sec. 5, we will consider the case where only channel fading statistics are available.

4.1 The optimum LR-based fusion rule

Using the multi-hop fusion model as described in Fig. 2 and Sec. 3, we can derive the LR at the fusion center.

First we introduce the notion of composite local performance indices, $P_{dk}^{(c)}$ and $P_{fk}^{(c)}$, defined as

$$\begin{aligned} P_{dk}^{(c)} &= P(u_k^{M_k} = 1|H_1) \\ P_{fk}^{(c)} &= P(u_k^{M_k} = 1|H_0) \end{aligned}$$

They are the probabilities of declaring H_1 at the last relay when the true hypothesis is H_1 and H_0 , respectively. This is different from local performance indices, P_{dk} and P_{fk} .

Given $P_{dk}^{(c)}$ and $P_{fk}^{(c)}$, the LR can be written as

$$\begin{aligned}\Lambda_1 &= \frac{f(\mathbf{y}|H_1)}{f(\mathbf{y}|H_0)} \\ &= \prod_{k=1}^K \frac{P_{dk}^{(c)} e^{-\frac{(y_k - h_k^{M_k})^2}{2\sigma^2}} + (1 - P_{dk}^{(c)}) e^{-\frac{(y_k + h_k^{M_k})^2}{2\sigma^2}}}{P_{fk}^{(c)} e^{-\frac{(y_k - h_k^{M_k})^2}{2\sigma^2}} + (1 - P_{fk}^{(c)}) e^{-\frac{(y_k + h_k^{M_k})^2}{2\sigma^2}}}\end{aligned}\quad (7)$$

where the assumption of conditional independence of local decisions has been employed.

Using the model specified in Sec. 3, the two parameters $P_{dk}^{(c)}$ and $P_{fk}^{(c)}$ can be derived as follows. First, define

$$\begin{aligned}P_{1k}^{M_k} &= P(u_k^{M_k} = 1 | u_k^0 = 1, H_1) \\ &= P(u_k^{M_k} = 1 | u_k^0 = 1) \\ P_{2k}^{M_k} &= P(u_k^{M_k} = 1 | u_k^0 = -1, H_1) \\ &= P(u_k^{M_k} = 1 | u_k^0 = -1)\end{aligned}$$

Hence,

$$P_{dk}^{(c)} = P_{dk} P_{1k}^{M_k} + (1 - P_{dk}) P_{2k}^{M_k} \quad (8)$$

$$P_{fk}^{(c)} = P_{fk} P_{1k}^{M_k} + (1 - P_{fk}) P_{2k}^{M_k} \quad (9)$$

$P_{1k}^{M_k}$ and $P_{2k}^{M_k}$ can be recursively determined as in the first part of Appendix A. Then, based on Eqs. (8,9), we can obtain $P_{dk}^{(c)}$ and $P_{fk}^{(c)}$. Thus the optimum LR test can be constructed accordingly.

4.2 Suboptimum fusion rules with known channel envelope

Implementing the optimum LR-based fusion rule using Λ_1 as given in Eq. (7) requires complete channel knowledge of all the hops and the composite local performance indices. To relieve the above requirements, we propose two alternatives as low and high channel SNR approximations to the optimum LR-based fusion rule. Consider the high SNR case first.

At high SNR, i.e., $\sigma^2 \rightarrow 0$, it is easy to show that $P_{1k}^{M_k} \approx 1$ and $P_{2k}^{M_k} \approx 0$. Thus, based on Eqs. (8,9), $P_{dk}^{(c)}$ and $P_{fk}^{(c)}$ with known channel envelope for a multi-hop WSN can be approximated as: $P_{dk}^{(c)} \approx P_{dk}$ and $P_{fk}^{(c)} \approx P_{fk}$. This leads to the following result.

Proposition 1 *In the high SNR case, the log likelihood ratio (LLR) with known channel envelope for a multi-hop WSN can be approximated as the Chair-Varshney fusion rule:*

$$\Lambda_3 \triangleq \log \Lambda_1 \approx \sum_{y_k < 0} \log \left(\frac{1 - P_{dk}}{1 - P_{fk}} \right) + \sum_{y_k > 0} \log \left(\frac{P_{dk}}{P_{fk}} \right) \quad (10)$$

Eq. (10) is precisely the decision statistic of the optimum fusion rule derived in [10]. We notice that the high channel SNR approximation for the multi-hop case is the same as the one derived in [6] for the single hop case. Intuitively, at

high SNR, each relay node tends to make the right decision, and thus can be ignored in the fusion rule design.

Next, we will give the low SNR approximation of Λ_1 .

First we present the low SNR approximation for the composite performance indices.

Lemma 1 *At low SNR, $P_{dk}^{(c)}$ and $P_{fk}^{(c)}$ with known channel envelope for a multi-hop WSN can be approximated as:*

$$P_{dk}^{(c)} \approx \frac{1}{2} + \frac{2^{M_k-1} \left(\prod_{m=0}^{M_k-1} h_k^m \right)}{(\sqrt{2\pi}\sigma)^{M_k}} (2P_{dk} - 1) \quad (11)$$

$$P_{fk}^{(c)} \approx \frac{1}{2} + \frac{2^{M_k-1} \left(\prod_{m=0}^{M_k-1} h_k^m \right)}{(\sqrt{2\pi}\sigma)^{M_k}} (2P_{fk} - 1) \quad (12)$$

Due to the limited space, we give the following proposition without proof.

Proposition 2 *In the low SNR case, the LLR with known channel envelope for a multi-hop WSN can be approximated as:*

$$\log \Lambda_1 \approx \sum_{k=1}^K (P_{dk} - P_{fk}) \frac{2^{M_k+1} \left(\prod_{m=0}^{M_k} h_k^m \right) y_k}{(\sqrt{2\pi}\sigma)^{M_k} \sigma^2} \quad (13)$$

To show this, we note that as $\sigma^2 \rightarrow \infty$, $e^{-\frac{2y_k h_k^{M_k}}{\sigma^2}} \rightarrow 1$ and can be approximated by the first order Taylor series expansion, i.e., $e^{-\frac{2y_k h_k^{M_k}}{\sigma^2}} \approx 1 - \frac{2y_k h_k^{M_k}}{\sigma^2}$. Based on (7), the LR is then approximated as, for large σ^2 ,

$$\Lambda_1 \approx \prod_{k=1}^K \frac{P_{dk}^{(c)} + (1 - P_{dk}^{(c)}) \left(1 - \frac{2y_k h_k^{M_k}}{\sigma^2} \right)}{P_{fk}^{(c)} + (1 - P_{fk}^{(c)}) \left(1 - \frac{2y_k h_k^{M_k}}{\sigma^2} \right)} \quad (14)$$

$$= \prod_{k=1}^K \frac{1 - (1 - P_{dk}^{(c)}) \frac{2y_k h_k^{M_k}}{\sigma^2}}{1 - (1 - P_{fk}^{(c)}) \frac{2y_k h_k^{M_k}}{\sigma^2}} \quad (15)$$

Taking logarithm on both sides of (15) and using the fact that $\log(1+x) \approx x$ when $x \rightarrow 0$, we have

$$\begin{aligned}\log \Lambda_1 &\approx - \sum_{k=1}^K (1 - P_{dk}^{(c)}) \frac{2y_k h_k^{M_k}}{\sigma^2} + \sum_{k=1}^K (1 - P_{fk}^{(c)}) \frac{2y_k h_k^{M_k}}{\sigma^2} \\ &= \sum_{k=1}^K (P_{dk} - P_{fk}) \frac{2^{M_k+1} \left(\prod_{m=0}^{M_k} h_k^m \right) y_k}{(\sqrt{2\pi}\sigma)^{M_k} \sigma^2}\end{aligned}\quad (16)$$

Assuming that all the local performance indices are identical, and neglecting the constant term that does not affect the detection performance, we can rewrite (13) as follows.

$$\begin{aligned}\log \Lambda_1 &\approx \sum_{k=1}^K \left(\frac{2}{\sqrt{2\pi}\sigma} \right)^{M_k} \left(\prod_{m=0}^{M_k} h_k^m \right) y_k = \sum_{k=1}^K h_k^{(c)} y_k \\ \Lambda_4 &\triangleq \sum_{k=1}^K h_k^{(c)} y_k\end{aligned}\quad (17)$$

where the composite channel envelope $h_k^{(c)} = \left[\prod_{m=0}^{M_k-1} \left(\frac{2}{\sqrt{2\pi}} \frac{h_k^m}{\sigma} \right) \right] h_k^{M_k}$.

Λ_4 is similar to the MRC statistic derived as a low SNR approximation for the single hop case in [6], except that the weighting function is the composite channel envelope $h_k^{(c)}$, which involves the product of weighted SNRs (with weight $\frac{2}{\sqrt{2\pi}}$) of all the hops except the last hop. Since $\frac{2}{\sqrt{2\pi}} < 1$, the fusion rule using Λ_4 deemphasizes those sensors with more hops in the low SNR regime.

5 Fusion rules with known channel fading statistics

In this section, we derive the optimum LR-based fusion rule assuming that only channel fading statistics are available. That is, we know the pdf of the Rayleigh fading channels but have no knowledge of the instantaneous channel gains. We also give two alternatives to this optimum LR-based fusion rule for the low and high channel SNR cases.

5.1 The optimum LR-based fusion rule with known channel fading statistics

Denote Λ_2 as the LR that corresponds to the case when only channel fading statistics are known. As in Sec. 4.1, $P_{1k}^{M_k}$ and $P_{2k}^{M_k}$ can be recursively determined as in the second part of Appendix A. Then using Eqs. (8,9), we can determine $P_{dk}^{(c)}$ and $P_{fk}^{(c)}$. The following theorem gives the form of Λ_2 .

Theorem 2 *The LR with known channel fading statistics and composite local performance indices for a multi-hop WSN is:*

$$\Lambda_2 = \prod_{k=1}^K \frac{1 + [P_{dk}^{(c)} - Q(ry_k)]\sqrt{2\pi}ry_k e^{\frac{(ry_k)^2}{2}}}{1 + [P_{fk}^{(c)} - Q(ry_k)]\sqrt{2\pi}ry_k e^{\frac{(ry_k)^2}{2}}} \quad (18)$$

where $r = \frac{1}{\sigma\sqrt{1+2\sigma^2}}$ and $Q(\cdot)$ is the complementary distribution function of a standard Gaussian random variable.

Proof.

Similar to the result in [7], we have:

$$\begin{aligned} f(y_n | u_n^{M_n} = 1, H_1) &= \frac{2\sigma}{\sqrt{2\pi}(1+2\sigma^2)} e^{-\frac{y_n^2}{2\sigma^2}} \times \\ &\quad \left[1 + Q(-ry_n)\sqrt{2\pi}ry_n e^{\frac{(ry_n)^2}{2}} \right] \\ f(y_n | u_n^{M_n} = -1, H_1) &= \frac{2\sigma}{\sqrt{2\pi}(1+2\sigma^2)} e^{-\frac{y_n^2}{2\sigma^2}} \times \\ &\quad \left[1 - Q(ry_n)\sqrt{2\pi}ry_n e^{\frac{(ry_n)^2}{2}} \right] \\ r &= \frac{1}{\sigma\sqrt{1+2\sigma^2}} \end{aligned}$$

Then:

$$f(y_n | H_1) = \sum_{u_n^{M_n}} f(y_n | u_n^{M_n}, H_1) P(u_n^{M_n} | H_1)$$

$$\begin{aligned} &= \frac{2\sigma}{\sqrt{2\pi}(1+2\sigma^2)} e^{-\frac{y_n^2}{2\sigma^2}} \\ &\quad \times \left[1 + (P_{dn}^{(c)} - Q(ry_n))\sqrt{2\pi}ry_n e^{\frac{(ry_n)^2}{2}} \right] \end{aligned}$$

Similarly,

$$\begin{aligned} f(y_n | H_0) &= \frac{2\sigma}{\sqrt{2\pi}(1+2\sigma^2)} e^{-\frac{y_n^2}{2\sigma^2}} \\ &\quad \times \left[1 + (P_{fn}^{(c)} - Q(ry_n))\sqrt{2\pi}ry_n e^{\frac{(ry_n)^2}{2}} \right] \end{aligned}$$

Thus:

$$\begin{aligned} \Lambda_2 &= \frac{f(\mathbf{y} | H_1)}{f(\mathbf{y} | H_0)} \\ &= \prod_{n=1}^N \frac{1 + [P_{dn}^{(c)} - Q(ry_n)]\sqrt{2\pi}ry_n e^{\frac{(ry_n)^2}{2}}}{1 + [P_{fn}^{(c)} - Q(ry_n)]\sqrt{2\pi}ry_n e^{\frac{(ry_n)^2}{2}}} \quad (20) \end{aligned}$$

Q.E.D.

We notice that the optimum fusion rules based on (7) and (18) have a similar form as the ones derived in [6] and [7] for the single hop case. The difference is that instead of using local performance indices P_{dk} and P_{fk} , we use the composite local performance indices $P_{dk}^{(c)}$ and $P_{fk}^{(c)}$ which are functions of local performance indices and channel SNRs corresponding to the relay links.

The optimum fusion rule Λ_2 as in (18) for known channel fading statistics requires knowledge of the composite local performance indices, which involve the number of relays for each local sensor and channel parameters. Next, we derive several suboptimum fusion rules that alleviate the requirement of *a priori* information.

5.2 Suboptimum fusion rules with known channel fading statistics

Again, we start by considering the low and high channel SNR approximations of the optimum LR fusion rule Λ_2 .

At high SNR, $P_{dk}^{(c)}$ and $P_{fk}^{(c)}$ with known channel fading statistics for a multi-hop WSN can be approximated as: $P_{dk}^{(c)} \approx P_{dk}$ and $P_{fk}^{(c)} \approx P_{fk}$. This leads to the following result.

Proposition 3 *In the high SNR case, the LLR with known channel fading statistics for a multi-hop WSN approaches the Chair-Varshney fusion rule:*

$$\log \Lambda_2 \approx \sum_{y_k < 0} \log \left(\frac{1 - P_{dk}}{1 - P_{fk}} \right) + \sum_{y_k > 0} \log \left(\frac{P_{dk}}{P_{fk}} \right) \quad (21)$$

This high channel SNR approximation with known channel fading statistics for the multi-hop case is the same as that in Sec. 4 for the known channel envelope case.

Next, we will give the low SNR approximation of Λ_2 .

Lemma 2 *At low SNR, $P_{dk}^{(c)}$ and $P_{fk}^{(c)}$ with known channel fading statistics for a multi-hop WSN can be approximated*

as

$$P_{dk}^{(c)} \approx \frac{1}{2} + \frac{1}{(\sqrt{2}\sigma)^{M_k}} (P_{dk} - \frac{1}{2}) \quad (22)$$

$$P_{fk}^{(c)} \approx \frac{1}{2} + \frac{1}{(\sqrt{2}\sigma)^{M_k}} (P_{fk} - \frac{1}{2}) \quad (23)$$

Given the low SNR approximation of the composite performance indices, we have

Proposition 4 *At low SNR, the LLR with known channel fading statistics for a multi-hop WSN can be approximated as:*

$$\log \Lambda_2 \approx \sum_{k=1}^K \frac{\sqrt{2\pi}(p_{dk} - P_{fk})r}{(\sqrt{2}\sigma)^{M_k}} y_k \quad (24)$$

If all the sensors have the same local performance, and we neglect any constant term that does not affect detection performance, then,

$$\log \Lambda_2 \approx \sum_{k=1}^K \frac{1}{(\sqrt{2}\sigma)^{M_k}} y_k \triangleq \hat{\Lambda}_5 \quad (25)$$

The suboptimum fusion rule based on (25) again deemphasizes those sensors with more hops because of the weight factor $\frac{1}{(\sqrt{2}\sigma)^{M_k}}$ which is < 1 in the low SNR regime (large σ^2).

If all the sensors also have the same number of relays, i.e., M_k are the same, then,

$$\log \Lambda_2 \approx \alpha \sum_{k=1}^K y_k \quad (26)$$

with $\alpha = \frac{\sqrt{2\pi}(P_d - P_f)r}{(\sqrt{2}\sigma)^{M_k}}$, which is a constant.

Notice that (26) is analogous to the EGC statistic. Since the constant does not affect detection performance,

$$\log \Lambda_2 \approx \frac{1}{K} \sum_{k=1}^K y_k \triangleq \Lambda_5 \quad (27)$$

This EGC form of the low SNR approximation of the fusion rule for the multi-hop case is similar to that obtained in [7] for the single hop case .

6 Simulation results

In this section, we compare the performance of the fusion rules proposed in Secs. 4 and 5 using simulation.

For ease of SNR calculation we assume that all the channels have Rayleigh fading statistics with $E[(h_k^i)^2] = 1, k = 1, 2, \dots, K, i = 0, 1, \dots, M_k$. Binary decisions $u_k^i \in \{+1, -1\}, k = 1, 2, \dots, K, i = 0, 1, \dots, M_k$, are made at the local sensors and the relay nodes. We also assume that all the local sensors have identical performance indices P_d and P_f , and the same number of hops. Specifically, we set $P_f = 0.05$ and $P_d = 0.5$ with the total number of local sensors K equal to 8. Here we consider the number of hops

up to 3. In all the following figures, “LR” corresponds to the optimum fusion rule Λ_1 when complete channel knowledge is available; “LR with channel statistics” corresponds to the optimum fusion rule Λ_2 assuming only knowing the channel statistics, and “MRC” refers to the MRC-like statistic Λ_4 .

Figs. 3- 5 present the receiver operating characteristic (ROC) curves at 5dB channel SNR with the number of hops of 1, 2, and 3, respectively. The optimum LR-based fusion rule (Λ_1) when complete channel knowledge is available gives the uniformly most powerful detection performance; the fading statistic based LR fusion rule (Λ_2) gives slightly worse performance than Λ_1 . We notice that the EGC statistic Λ_5 performs better than the MRC-like statistic Λ_4 and the Chair-Varshney rule at 5 dB while requiring the least amount of prior information.

Figs. 6- 8 show the curves for the probability of detection P_d as a function of channel SNR for various fusion rules with the number of hops of 1, 2, and 3, respectively. We use a constant system false alarm rate of $P_{f0} = 0.01$. We see that at low SNR and high SNR, the MRC-like statistic Λ_4 and the Chair-Varshney rule can approach the optimum LR fusion rule Λ_1 , respectively; while EGC and Chair-Varshney statistic can approach the optimum LR fusion rule Λ_2 at low and high SNR, respectively. We also observe that at high SNR the MRC statistics Λ_4 performs much worse than the other alternatives. An intuitive explanation is that at high SNR, because of the hop dependent weight function h_k^c , those sensors with more hops are given more weight. In other words, those sensors with less hops are deemphasized because of the unequal weight function. Yet we know that at high SNR all the relay links tend to give reliable estimation hence applying unequal weights provides worse performance. Further, EGC scheme Λ_5 provides better performance than the MRC-like statistic Λ_4 and the Chair-Varshney rule over the moderate SNR range.

7 Conclusions

In this paper, we have designed fusion rules for binary decisions transmitted over multi-hop wireless channels with Rayleigh fading and additive Gaussian noise. We derived the optimum LR-based fusion rule for two cases: with complete channel knowledge and with the knowledge of channel fading statistics. For both cases we showed that the Chair-Varshney fusion rule approaches the optimum LR fusion rule at high channel SNR, while at low channel SNR the two LR fusion rules reduce to different forms of weighted sums of the fading channel outputs. Both low SNR suboptimum fusion rules deemphasize those sensors with more hops. Specifically, with complete channel knowledge, low channel SNR approximation leads to a MRC-like scheme and the weight function is a product of all the link SNRs along the relay path. With known channel fading statistics, the weight involves local performance indices and channel parameters. Under certain conditions, this low channel SNR approximation is equivalent to a simple EGC form. Furthermore, we demonstrated that EGC outperforms the MRC-like scheme and the Chair-Varshney

rule over the moderate SNR range with the least amount of prior information.

Our work is based on the assumption that the target is not observed directly by the relay nodes. Each relay node makes a binary decision based on its noisy input and sends it to its next level relay. This may not be the optimum relay strategy. Further research will focus on the signaling scheme design for the relay node and how the signaling will affect the fusion rule.

A Derivation of $P_{1k}^{M_k}$ and $P_{2k}^{M_k}$

Define

$$P_{1k}^i = P(u_k^i = 1 | u_k^0 = 1, H_1) \quad (28)$$

$$P_{2k}^i = P(u_k^i = 1 | u_k^0 = -1, H_1) \quad (29)$$

1. Given the assumption of known channel envelope, $P_{1k}^{M_k}$ can be recursively determined as follows.

$$P_{1k}^1 = P(h_k^0 + n_k^0 > 0) = 1 - Q\left(\frac{h_k^0}{\sigma}\right)$$

$$P_k^i = P(h_k^i + n_k^i > 0) = 1 - Q\left(\frac{h_k^{i-1}}{\sigma}\right)$$

$$P_{1k}^{m+1} = P_k^{m+1} P_{1k}^m + (1 - P_k^{m+1})(1 - P_{1k}^m)$$

$$P_{1k}^{M_k} = P_k^{M_k} P_{1k}^{M_k-1} + (1 - P_k^{M_k})(1 - P_{1k}^{M_k-1})$$

$P_{2k}^{M_k}$ can be similarly recursively determined. In fact, because each hop can be viewed as a binary symmetric channel, we can show that $P_{2k}^{M_k} = 1 - P_{1k}^{M_k}$.

2. Given the assumption of known channel fading statistics, we have

$$P_{1k}^1 = \frac{1}{2} + \frac{1}{2\sqrt{1+2\sigma^2}} \quad (30)$$

$$P_k^i = \frac{1}{2} + \frac{1}{2\sqrt{1+2\sigma^2}} \quad (31)$$

Then, based on Eqs. (30, 30), we can recursively determine $P_{1k}^{M_k}$.

$P_{2k}^{M_k}$ can be obtained in a similar fashion. Alternatively, we still have $P_{2k}^{M_k} = 1 - P_{1k}^{M_k}$.

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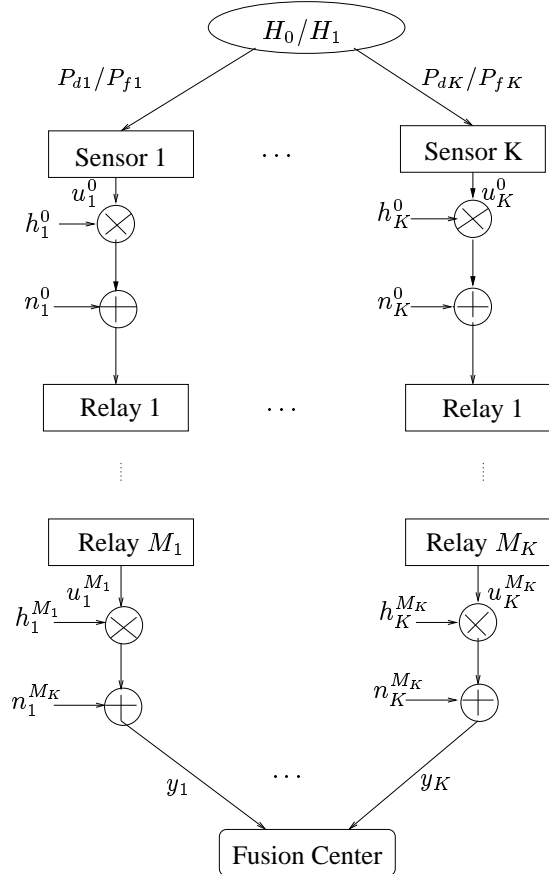


Fig. 2: Multi-hop parallel fusion model in the presence of fading and noisy channels between local sensors and the fusion center

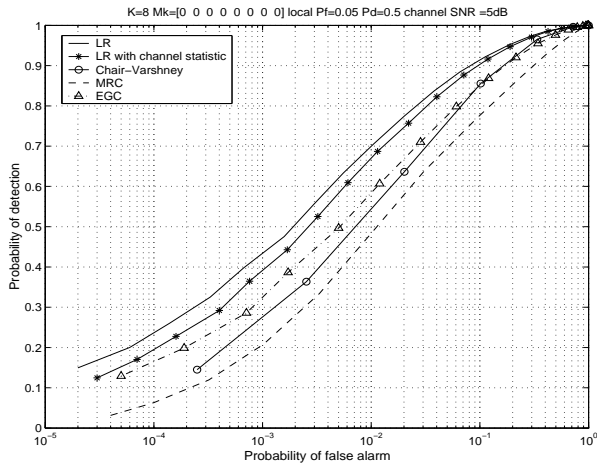


Fig. 3: ROC curves for various fusion rules for the Rayleigh fading channel with one hop and average channel SNR=5dB

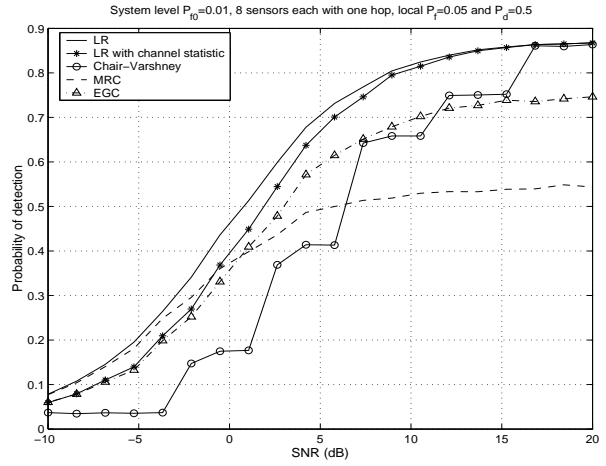


Fig. 6: Probability of detection as a function of channel SNR for various fusion rules for the Rayleigh fading channel with one hop.

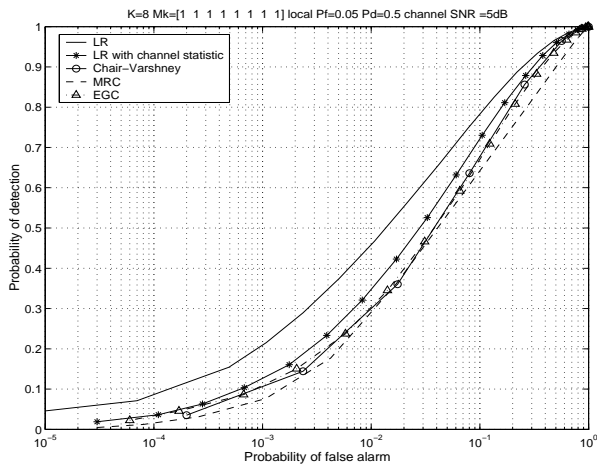


Fig. 4: ROC curves for various fusion rules for the Rayleigh fading channel with two hops and average channel SNR=5dB

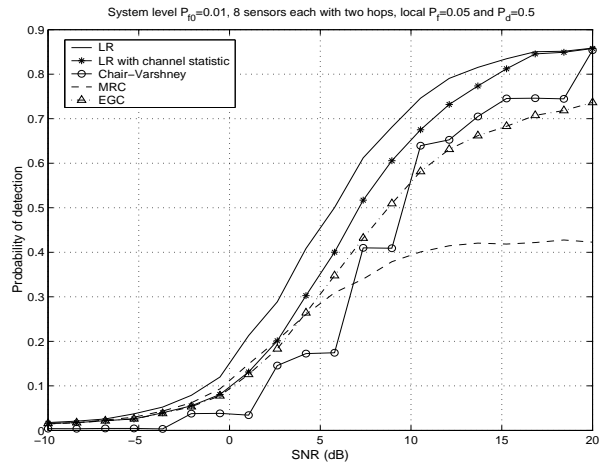


Fig. 7: Probability of detection as a function of channel SNR for various fusion rules for the Rayleigh fading channel with two hops.

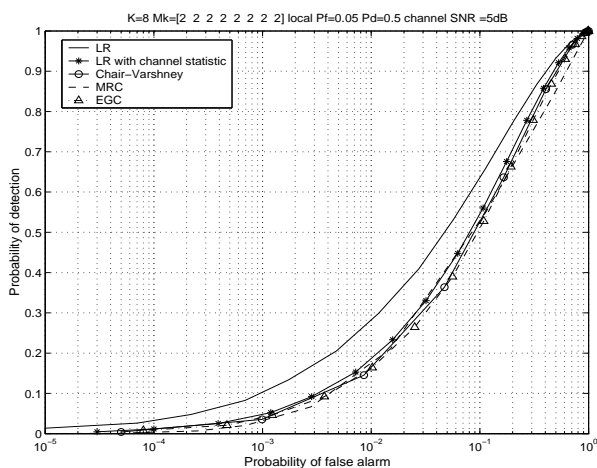


Fig. 5: ROC curves for various fusion rules for the Rayleigh fading channel with three hops and average channel SNR=5dB

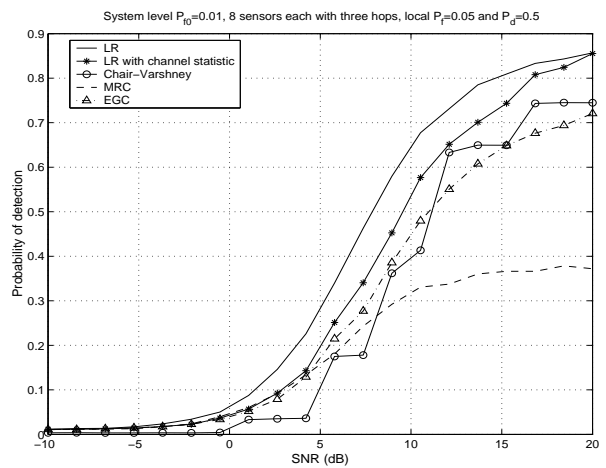


Fig. 8: Probability of detection as a function of channel SNR for various fusion rules for the Rayleigh fading channel with three hops.