Sensor Management with Non-Ideal Sensor Dynamics

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Abstract – *We describe a theoretically foundational but potentially practical control-theoretic basis for multisensor-multitarget sensor management using a comprehensive, intuitive, system-level Bayesian paradigm based on random set theory. We focus on mobile sensors whose states are observed indirectly by internal actuator sensors. We determine optimal controls (future sensor states) using a "probabilistically natural" sensor management objective function, the posterior expected number of targets (PENT). PENT is constructed using a new "maxi-PIMS" optimization strategy to hedge against unknowable future observation-collections. It is used in conjunction with the PHD or MHC approximate multitarget filters.*

Keywords: sensor management, random sets, control theory.

1 Introduction

Sensor management is inherently an *optimal nonlinear control problem*. Sensor management differs from standard control applications, however, in that it is also inherently a *stochastic multi-object problem*. It involves *randomly varying sets of targets*, *randomly varying sets of sensors/sources*, *randomly varying sets of collected data*, and *randomly varying sets of sensor-carrying platforms*.

This paper summarizes a theoretically foundational but potentially practical control-theoretic basis for multisensor-multitarget sensor management using the following system-level, Bayesian paradigm:

- Model all sensors and targets as a single joint dynamically evolving multi-object stochastic system;
- propagate the state of this system using a multisensor-multitarget Bayes filter;
- apply objective functions that express global probabilistic goals for sensor management;
- apply optimization strategies that hedge against the inherent unknowability of future observation-collections;
- devise principled approximations of this general (but usually intractable) formulation.

The last step is crucial and difficult: devise principled, potentially tractable: (1) multitarget filters; (2) global objective functions; and (3) optimization strategies. It requires *finite-set statistics* (FISST) [4, 6, 10, 17, 19], the novel random-set version of point process theory that is the subject of an invited keynote lecture and paper [18] at this conference.

In recent years, however, a few partisans have claimed that a so-called "plain-vanilla Bayesian approach" suffices as down-to-earth, general "first principles" for Bayes multitarget filtering and sensor management. FISST is, therefore, mathematical "obfuscation." But as we argue in our keynote paper [18] and elsewhere [5, 7, 10, 19], the "plain-vanilla" partisans have manufactured a spurious appearance of simplicity and progress, by promoting a succession of algorithms that are certainly "straightforward" but also afflicted by inherent—and less than candidly acknowledged—computational "logjams."

By way of contrast, we have chosen to investigate the deep structure of multitarget filtering and sensor management, with the aim of developing principled approximation strategies. Our work currently encompasses the following aspects of sensor management:

- targets of current or potential tactical interest [14, 20];
- multistep look-ahead (control of sensor resources throughout a future time-window) [14, 22];
- sensors with non-ideal dynamics, including sensors residing on moving platforms such as UAVs [16];
- sensors whose states are observed indirectly by internal actuator sensors [14]; and
- possible communication interference [14].

Our approach also addresses a more subtle issue: *the impossibility of deciding between an infinitude of plausible objective functions*, by concentrating on "probabilistically natural" sensor management goals. Our objective function, the *posterior expected number of targets* (PENT), is constructed using a new optimization strategy, "maxi-PIMS," that optimizes the likelihood of collecting the *predicted ideal measurement-set* (PIMS). Intuitively speaking, in a PIMS there are no false alarm/clutter observations, every target in the FoV generates an observation, and target-generated observations are noise-free.

The PENT objective function is used in conjunction with approximate multitarget filters: the probability hypothesis density (PHD) filter or the multi-hypothesis correlator (MHC) filter. Preliminary simulations using PENT with an MHC filter have demonstrated good sensor management behavior [3].

Full details of the approach can be found in [14]. This paper is a summary, but with communications assumed to be ideal. Its purpose is not only to describe our progress, but also to sketch the basic concepts of principled—as opposed to "plain-vanilla Bayesian"—approximation.

1.1 Organization of the paper

We summarize the mathematical foundations required to understand the paper in section 2. The multitarget Bayes filter and its approximations, the PHD filter and MHC filter, are discussed in section 3. Our core approach to sensor management is summarized in section 4. The new "maxi-PIMS" optimization strategy is described in section 5. It is used to derive formulas for our primary objective function, the posterior expected number of targets (PENT) in section 6, assuming that sensor dynamics are ideal. Section 7 describes the extension of PENT to sensors that are dynamically evolving and whose states are observable only through the mediation of internal actuator sensors. Conclusions may be found in section 8.

2 Mathematical preliminaries

2.1 States and observations

The states of a multitarget system have the form $X = \{\mathbf{x}_1,...,\mathbf{x}_n\}$ where the number n and states $\mathbf{x}_1,...,\mathbf{x}_n$ of the targets are random. Measurements have the form $Z = \{\mathbf{z}_1,...,\mathbf{z}_m\}$ where the number m is random as well as the individual measurements $\mathbf{z}_1,...,\mathbf{z}_m$ themselves.

Sensors also have state vectors $\mathbf{x}^{*i}$ where i denotes the sensor tag (identifier) of the i'th sensor. If only one sensor is present we will ignore the tag and write $\mathbf{x}^*$.

2.2 Integration

Because states and measurements can vary randomly in number, integration must account for this fact. Let $f(X)$ be a real-valued function of a finite-set variable X. Then the *set integral* [4, 6, 10, 17, 19] is defined by

$$\int_S f(X)\delta X = f(\emptyset) + \sum_{n=0}^{\infty} \frac{1}{n!} \int_{S^n} f(\{\mathbf{x}_1,...,\mathbf{x}_n\}) d\mathbf{x}_1 \cdots d\mathbf{x}_n \quad (1)$$

If $\int f(X)\delta X = 1$ then $f(X)$ is a *multitarget probability density* [4, 6, 10, 17, 19].

2.3 Probability generating functionals

Given a random finite set Ψ of vectors $\mathbf{y}$ in some space $\boldsymbol{Y}$. Given any function of the form

$$h(\mathbf{y}) = h_0(\mathbf{y}) + w_1\delta_{\mathbf{w}_1}(\mathbf{y}) + ... + w_m\delta_{\mathbf{w}_m}(\mathbf{y})$$

where $0 \leq h_0(\mathbf{x}) \leq 1$ has no units of measurement; where $\delta_{\mathbf{w}}(\mathbf{y})$ is the Dirac delta; where $\mathbf{w}_1,\ldots,\mathbf{w}_m$ are distinct elements of $\boldsymbol{Y}$; and where $w_1,\ldots,w_m$ have the same units of measurement as the $\mathbf{w}_i$'s. Then the *probability generating functional* (p.g.fl.) of Ψ is:

$$G_\Psi[h] = \int h^Y f_\Psi(Y)\delta Y \quad (2)$$

(see [13, 12, 16] and pp. 141, 220 of [2]). Because $f_\Psi(\{\mathbf{y}_1,\ldots,\mathbf{y}_i,\ldots,\mathbf{y}_j,\ldots,\mathbf{y}_n\}) = 0$ whenever $\mathbf{y}_i = \mathbf{y}_j$ for $i \neq j$, the p.g.fl. is well-defined and finite-valued because undefined products of the form $\delta_{\mathbf{u}}(\mathbf{y})^2$ cannot occur.

The intuitive meaning of the p.g.fl. is as follows. Let $\boldsymbol{Y} = \boldsymbol{X}$ be single-target state space, $\Psi = \Xi$ a random finite subset of $\boldsymbol{X}$, and $0 \leq h(\mathbf{x}) \leq 1$, so that $h(\mathbf{x})$ can be interpreted as the field of view (FoV) of some sensor. Then $G_\Xi[h]$ is the probability that Ξ is contained in the FoV. Since $h(\mathbf{x})$ is also a fuzzy membership function on $\boldsymbol{X}$, $G_\Xi[h]$ is a generalization of the belief-mass function $\beta_\Xi(S) = G_\Xi[\mathbf{1}_S]$ from crisp sets S to fuzzy subsets h.

2.4 Functional derivatives of p.g.fl.'s

The *gradient derivative* (a.k.a. Frechét derivative) of a p.g.fl. $G[h]$ in the direction of the function g is

$$\frac{\partial G}{\partial g}[h] = \lim_{\varepsilon \to 0} \frac{G[h + \varepsilon \cdot g] - G[h]}{\varepsilon} \quad (3)$$

where for each h the functional $g \to \frac{\partial G}{\partial g}[h]$ is linear and continuous. Gradient derivatives obey the usual "turn the crank" rules of undergraduate calculus, e.g. sum rule, product rule, etc. The *functional derivatives* of $G[h]$ are gradient derivatives in the direction of Dirac deltas $g = \delta_{\mathbf{x}}$:

$$\frac{\delta^0 G}{\delta^0 \mathbf{x}^0}[h] = G[h], \qquad \frac{\delta G}{\delta \mathbf{x}}[h] = \frac{\partial G}{\partial \delta_{\mathbf{x}}}[h] \quad (4)$$

$$\frac{\delta G}{\delta X}[h] = \frac{\delta^n G}{\delta \mathbf{x}_1 \cdots \delta \mathbf{x}_n}[h] = \frac{\partial G}{\partial \delta_{\mathbf{x}}}[h] \quad (5)$$

for $X = \{\mathbf{x}_1,\ldots,\mathbf{x}_n\}$ with $\mathbf{x}_1,\ldots,\mathbf{x}_n$ distinct.

2.5 Probability hypothesis densities (PHDs)

For a random finite set Ψ of vectors $\mathbf{y}$ in $\boldsymbol{Y}$,

$$D_\Psi(\mathbf{y}) = \frac{\delta G_\Psi}{\delta \mathbf{y}}[1] = \int f_\Psi(\{\mathbf{y}\} \cup Y)\delta Y \quad (6)$$

is the *first-moment density* or *probability hypothesis density (PHD)* of Ψ. The PHD is characterized uniquely by the following property: Its integral in any region S of $\boldsymbol{Y}$ is the expected number of objects in that region: $\int_S D_\Psi(\mathbf{y})d\mathbf{y} = \mathrm{E}[\,|\Psi \cap S|\,]$. Note that

$$\frac{\delta \log G_\Psi}{\delta \mathbf{y}}[1] = \left[\frac{1}{G_\Psi[h]} \cdot \frac{\delta G_\Psi}{\delta \mathbf{y}}[h]\right]_{h=1} = D_\Psi(\mathbf{y}) \quad (7)$$

PHDs can be computed using the often simpler $\log G_\Psi[h]$.

3 Multitarget filtering

The general multitarget Bayes filter is described in section 3.1 and its reformulation in terms of p.g.fl.'s in section 3.2. Simpler filters are required to approximate it: the probability hypothesis density (PHD) filter (section 3.3) and the multi-hypothesis correlator (MHC) filter (section 3.5). The procedure for deriving the PHD filter, which is paradigmatic for later sections, is sketched in Section 3.4.

3.1 The multitarget Bayes filter

The general foundation for multisensor-multitarget detection, tracking, and identification is the following generalization of the recursive Bayes filter:

$$f_{k+1|k}(X \mid Z^{(k)}) = \int f_{k+1|k}(X \mid W) \cdot f_{k|k}(W \mid Z^{(k)})\delta W \quad (8)$$

$$f_{k+1|k+1}(X \mid Z^{(k+1)}) = \frac{f_{k+1}(Z_{k+1} \mid X) \cdot f_{k+1|k}(X \mid Z^{(k)})}{f_{k+1}(Z_{k+1} \mid Z^{(k)})} \quad (9)$$

where

$$f_{k+1}(Z \mid Z^{(k)}) = \int f_{k+1}(Z \mid X) \cdot f_{k+1|k}(X \mid Z^{(k)})\delta X$$

is the Bayes normalization factor and where

1) $f_k(Z|X)$ is the *multisensor, multitarget likelihood function* that describes the likelihood of observing the observation-set Z given that the targets have multitarget state-set X;
2) $f_{k+1|k}(X|W)$ is the *multitarget Markov transition density* that models interim motion, including target appearance and disappearance as well as individual target motion.

The multitarget Bayes filter is not the straightforward extension of the single-target Bayes filter that it appears to be [5, 7, 10, 19]. Its proper development requires FISST.

For the remainder of the paper we will abbreviate:

$$f_{k+1|k}(X) = f_{k+1|k}(X \mid Z^{(k)}) \tag{10}$$

$$f_{k+1|k+1}(X) = f_{k+1|k+1}(X \mid Z^{(k+1)}) \tag{11}$$

3.2 p.g.fl. form of the multitarget filter

Our approach is based on reformulation of Eqs. (8) and (9) in terms of p.g.fl.'s. This reformulation opens the way to systematic approach to approximation [7, 14].

· *p.g.fl. representation of Eq. (8)*: The multitarget prediction integral can be rewritten as [12, 14, 16]:

$$G_{k+1|k}[h] = \int G_{k+1|k}[h \mid X] \cdot f_{k|k}(X)\delta X \tag{12}$$

where

$$G_{k+1|k}[h \mid X] = \int h^Y f_{k+1|k}(Y \mid X)\delta Y \tag{13}$$

· *p.g.fl. version of Eq. (9)*: Define $f_{k+1}[g,h]$ by

$$F_{k+1}[g,h] = \int g^Z h^X f_{k+1}(Z \mid X) f_{k+1|k}(X)\delta X \delta Z \tag{14}$$

Then Eq. (9) can be equivalently written as [12, 14, 16]:

$$G_{k+1|k+1}[h] = \frac{\frac{\delta F_{k+1}}{\delta Z_{k+1}}[0,h]}{\frac{\delta F_{k+1}}{\delta Z_{k+1}}[0,1]} \tag{15}$$

3.3 PHD approximate multitarget filter

The PHD was defined in section 2.5. This section summarizes an approximation of the multitarget Bayes filter by a multitarget filter that propagates PHDs in place of multitarget posterior distributions.

· *PHD Predictor Equation*: $D_{k|k}(\mathbf{x})$ can be extrapolated to the next time-step using [12, 13]:

$$D_{k+1|k}(\mathbf{x}) = b_{k+1|k}(\mathbf{x}) + \int s_{k+1|k}(\mathbf{w}) f_{k+1|k}(\mathbf{x} \mid \mathbf{w}) D_{k|k}(\mathbf{w}) d\mathbf{w} \tag{16}$$

Here $f_{k+1|k}(\mathbf{x}|\mathbf{w})$ is the Markov transition for single targets; $s_{k+1|k}(\mathbf{w})$ is the probability that a target with state $\mathbf{w}$ at time-step k will survive into time-step k+1; and $b_{k+1|k}(\mathbf{x}) = \int b_{k+1|k}(\{\mathbf{x}\} \cup X)\delta X$ is the PHD of $b_{k+1|k}(X)$, where $b_{k+1|k}(X)$ is the probability that targets with state-set X will appear in the scene.

· *PHD Corrector Equation*: Assume that probability of detection and likelihood for the sensor are

$$p_D(\mathbf{x}) = p_D(\mathbf{x}, \mathbf{x}^*_{k+1}), \quad L_{\mathbf{z}}(\mathbf{x}) = f_{k+1}(\mathbf{z} \mid \mathbf{x}, \mathbf{x}^*_{k+1})$$

respectively. Assume that the observations are corrupted by a Poisson false alarm process

$$\lambda = \lambda_{k+1}, \quad c(\mathbf{z}) = c_{k+1}(\mathbf{z})$$

Let $Z_{k+1} = \{\mathbf{z}_1,\ldots, \mathbf{z}_m\}$ be the new observation-set. Then $D_{k+1|k}(\mathbf{x})$ of $G_{k+1|k}[h]$ can be updated using [12, 13]:

$$D_{k+1|k+1}(\mathbf{x}) = \begin{pmatrix} 1 - p_D(\mathbf{x}) \\ + \sum_{i=1}^{m} \frac{p_D(\mathbf{x}) L_{\mathbf{z}_i}(\mathbf{x})}{\lambda c(\mathbf{z}_i) + D_{k+1|k}[p_D L_{\mathbf{z}_i}]} \end{pmatrix} D_{k+1|k}(\mathbf{x}) \tag{17}$$

where $D_{k+1|k}[h] = \int h(\mathbf{x})\, D_{k+1|k}(\mathbf{x}) d\mathbf{x}$.

3.4 Derivation of PHD corrector equation

This section sketches the derivation of the PHD corrector Eq. (10), which is paradigmatic for computing the PENT objective function. By Eq. (7) $D_{k+1|k+1}(\mathbf{x})$ can be computed as a first functional derivative of log $G_{k+1|k+1}[h]$. But $G_{k+1|k+1}[h]$ can be computed from $F_{k+1}[g,h]$ of Eq. (14) using Eq. (15). Given the assumptions of the measurement model of section 3.3, $F_{k+1}[g,h]$ can be written in terms of $G_{k+1|k}[h]$:

$$F_{k+1}[g,h] = e^{-\lambda + \lambda c_g} \cdot G_{k+1|k}[h(1 - p_D + p_D p_g)] \tag{18}$$

where

$$c_g = \int g(\mathbf{z}) \cdot c_{k+1}(\mathbf{z}) d\mathbf{z}$$

$$p_g(\mathbf{x}) = \int g(\mathbf{z}) \cdot f_{k+1}(\mathbf{z} \mid \mathbf{x}, \mathbf{x}^*_{k+1}) d\mathbf{z}$$

Assume that the predicted p.g.fl. is approximately Poisson

$$G_{k+1|k}[h] \cong \exp\left(-N_{k+1|k} + D_{k+1|k}[h]\right) \tag{19}$$

where $N_{k+1|k} = \int D_{k+1|k}(\mathbf{x}) d\mathbf{x}$. Then from Eq. (18) we get

$$F_{k+1}[g,h] \cong \exp\left(-\lambda - N_{k+1|k} + \lambda c_g + D_{k+1|k}[h(1 - p_D + p_D p_g)]\right) \tag{20}$$

From this it follows that

$$\frac{\delta F_{k+1}}{\delta Z}[g,h] = F_{k+1}[g,h] \cdot \prod_{\mathbf{z} \in Z} \left(\lambda c(\mathbf{z}) + D_{k+1|k}[h p_D L_{\mathbf{z}}]\right)$$

So, from Eq. (15) the posterior p.g.fl. is

$$G_{k+1|k+1}[h] = G_{k+1}[h] \cdot \prod_{\mathbf{z} \in Z} \frac{\lambda c(\mathbf{z}) + D_{k+1|k}[h p_D L_{\mathbf{z}}]}{\lambda c(\mathbf{z}) + D_{k+1|k}[p_D L_{\mathbf{z}}]} \tag{21}$$

where

$$G_{k+1}[h] = \exp\left(D_{k+1|k}[(h-1)(1-p_D)]\right) \tag{22}$$

Eq. (17) follows quickly by applying Eq. (7).

3.5 MHC approximate multitarget filter

Multi-hypothesis correlator (MHC) filters [1] are multitarget filters consisting of recursively alternating prediction and correction steps. At each step they produce a set of "hypotheses" as outputs, and probabilities that these hypotheses are valid representations of ground truth. Each hypothesis is a subset of a "track table" consisting of N tracks for some N. Each track has a linear-Gaussian probability distribution $f_j(\mathbf{x}) = N_{P_j}(\mathbf{x} - \mathbf{x}_j)$ where $\mathbf{x}_j$ is the estimated state of the track and P_j is its covariance matrix. The $f_1(\mathbf{x}),\ldots,f_N(\mathbf{x})$ are independent posterior densities constructed from a partition of the time-accumulated measurements.

Any track has a "track probability" q_j, which is the sum of the hypothesis probabilities of all hypotheses that contain that track; and which can be interpreted as the probability that the j'th track exists. The $q_1,\ldots, q_N$ are not necessarily independent because they do not arise from a unique partition of the accumulated measurements.

Nevertheless, the following equation for the predicted p.g.fl. can be assumed to be approximately true:

$$G_{k+1|k}[h] \cong \prod_{j=1}^{N} \left(1 - q_j + q_j f_j[h]\right) \tag{23}$$

where $f_j[h] = \int h(\mathbf{x})\, f_j(\mathbf{x})d\mathbf{x}$. In this case it is easy to see that the PHD of $G_{k+1|k}[h]$ is

$$D_{k+1|k}(\mathbf{x}) \cong \sum_{j=1}^{N} q_j f_j(\mathbf{x}) \tag{24}$$

and so we can use the even simpler Poisson approximation

$$G_{k+1|k}[h] \cong \exp\left(\sum_{j=1}^{N} q_j \left(1 - f_j[h]\right)\right) \tag{25}$$

This approximation allows us to apply to the MHC filter any formula that has been derived for the PHD filter. In any such formula, Eq. (24) can be used to substitute $\Sigma_j q_j f_j(\mathbf{x})$ wherever $D_{k+1|k}(\mathbf{x})$ occurs.

4 Multitarget sensor management

In this section we briefly review the system-level, control-theoretic approach to sensor management we have been pursuing for the last two years [11, 14-16, 20, 21, 22]. Our core approach, based on multitarget posterior distributions, was introduced in March 1996 [9] and slightly generalized in 1998 [8]. This section describes primarily our recent p.g.fl.-based refinement of it.

Assume that sensors are known and fixed in number. Let $\mathbf{x}^* = (\mathbf{x}^{*1},\ldots, \mathbf{x}^{*s})$ be the concatenation of their state vectors. Regard all targets and sensors as part of a single stochastically evolving system with joint state $(X, \mathbf{x}^*)$. Propagate this system using a joint recursive Bayes filter:

$$f_{k+1|k}(X,\mathbf{x}^*) = \int f_{k+1|k}(X,\mathbf{x}^* \mid W,\mathbf{w}^*) \cdot f_{k|k}(W,\mathbf{w}^*)\delta W$$

$$f_{k+1|k+1}(X,\mathbf{x}^*) = \frac{f_{k+1}(Z_{k+1} \mid X,\mathbf{x}^*) \cdot f_{k+1|k}(X,\mathbf{x}^*)}{f_{k+1}(Z_{k+1})}$$

where

$$f_{k+1}(Z) = \int f_{k+1}(Z \mid X,\mathbf{x}^*) \cdot f_{k+1|k}(X,\mathbf{x}^*)\delta X$$

The joint Markov transition

$$f_{k+1|k}(X,\mathbf{x}^*|W,\mathbf{w}^*) = f_{k+1|k}(X,\mathbf{x}^*|W,\mathbf{w}^*,\mathbf{u}_k)$$

actually depends on a joint control vector $\mathbf{u}_k = (\mathbf{u}_k^1,\ldots,\mathbf{u}_k^s)$ that influences the future joint sensor state $\mathbf{x}_{k+1}^*$. Consequently, the joint multisensor-multitarget posterior distributions $f_{k|k}(X,\mathbf{x}^*)$ implicitly depend on a time-sequence of control vectors. We have suppressed this dependence to keep notation simpler.

The p.g.fl.'s $G_{k|k}[h] = \int h^X f_{k|k}(X)\delta X$ and $G_{k+1|k}[h] = \int h^X f_{k+1|k}(X)\delta X$ contain the same information as $f_{k+1|k}(X)$ and $f_{k+1|k}(X)$. So we should instead concentrate on objective functions defined in terms of the posterior p.g.fl. $G_{k+1|k+1}[h]$. Since it depends on the unknown future observation-set Z_{k+1} we must hedge against this fact. We could produce a hedged p.g.fl. $\dot{G}_{k+1|k+1}[h]$ by taking the expectation of $G_{k+1|k+1}[h]$ over all observation-sets, but $\dot{G}_{k+1|k+1}[h]$ no longer has any dependence on the unknown control/future sensor state. The expectation of some nonlinear transform of $G_{k+1|k+1}[h]$ no longer has this problem, but will be intractable. Intractability results if we use a maxi-min approach, i.e. assume that the worst-possible observation-set has been collected. We studied a tractable "maxi-null" approach [8, 9, 11, 16, 21] that turned out to be too conservative. So we have devised the new "maxi-PIMS" strategy described in section 5. We produce a hedged posterior p.g.fl. $\dot{G}_{k+1|k+1}[h]$ using maxi-PIMS. The posterior expected number of targets (PENT) objective function $\dot{N}_{k+1|k+1}$ can be defined in terms of it.

Maximizing $\dot{N}_{k+1|k+1}$ results in *single-step look-ahead* sensor management—we select an optimal control only for the next time-step. In *multistep look-ahead* we determine optimal controls for a future time-window. Special techniques are required to deal with this [14, 22].

Suppose now that we approximate the multitarget filter using the PHD filter of section 3.3 or the MHC filter of section 3.5. Then the procedure sketched in section 3.4 can be used to derive closed-form formulas for objective functions defined in terms of $\dot{G}_{k+1|k+1}[h]$. Both such filters presume that the predicted p.g.fl. $G_{k+1|k}[h]$ has a simplified form. Therefore the p.g.fl. $F_{k+1}[g,h]$ has a simplified form and hence so does $\dot{G}_{k+1|k+1}[h]$. We can then derive closed-form formulas for PENT that are consistent with the approximate filters.

Once we have such formulas, they are used in conjunction with an approximate filter for sensor management. In single-step look-ahead we determine the next joint control vector (or joint sensor state) by optimizing the objective function. Collect the future observation-sets and use the predictor and corrector of the approximate filter to incorporate this new information. Repeat. Similarly for multistep look-ahead, except that controls/sensor states are chosen for a window, and the approximate filter is operated for all steps in that window.

5 "Maxi-PIMS" optimization-hedging

This section proposes a new potentially tractable optimization-hedging strategy. We begin assuming a single sensor, in which case the basic idea is this: choose the future FoV that will have the best chance of producing an "ideal" observation-set—i.e., no clutter observations, every target in the FoV generates an observation, and target-generated observations are noise-free.

The PIMS is introduced in section 5.1, the hedged posterior p.g.fl. $\dot{G}_{k+1|k+1}[h]$ in section 5.2, the hedged posterior PHD $\dot{D}_{k+1|k+1}(\mathbf{x})$ in section 5.3, and the single-step posterior expected number of targets (PENT) $\dot{N}_{k+1|k+1}(\mathbf{x}_{k+1}^*)$ in section 5.4. We extend the approach to the multisensor case in section 5.5.

5.1 Predicted ideal measurement-set (PIMS)

Assume that sensor likelihood functions have the form

$$L_{\mathbf{z}}(\mathbf{x}) = f_{k+1}(\mathbf{z} \mid \mathbf{x},\mathbf{x}_{k+1}^*) = f_{\mathbf{W}_{k+1}}(\mathbf{z} - \eta_{k+1}(\mathbf{x},\mathbf{x}_{k+1}^*)) \tag{26}$$

Abbreviate $\eta(\mathbf{x}) = \eta_{k+1}(\mathbf{x},\mathbf{x}_{k+1}^*)$. Begin by assuming that the future sensor FoV is a cookie-cutter: $p_D(\mathbf{x},\mathbf{x}_{k+1}^*) = \mathbf{1}_S(\mathbf{x})$ where $\mathbf{1}_S(\mathbf{x}) = 1$ if $\mathbf{x} \in S$ and $\mathbf{1}_S(\mathbf{x})$

= 0 otherwise. That is, an observation will be collected from a target if it is in the FoV, but not otherwise. Assume that some multitarget state estimation process has been used to estimate the number $\hat{n}$ and states $\hat{\mathbf{x}}_1,...,\hat{\mathbf{x}}_{\hat{n}}$ of the predicted tracks. Then an "ideal" noise- and clutter-free observation at time-step $k+1$ would be

$$Z_{k+1} = \bigcup\nolimits_{\hat{\mathbf{x}}_i \in S} \{\eta(\hat{\mathbf{x}}_i)\} \tag{27}$$

If the FoV is not a cookie cutter then we must account for the fact that p_D can have values between zero and one. Define the subset $S_a(p_D)$ of single-target state space by

$$S_a(p_D) = \{\mathbf{x}|\ a \le p_D(\mathbf{x})\} \tag{28}$$

where we abbreviate $p_D(\mathbf{x}) = p_D(\mathbf{x}, \mathbf{x}^*_{k+1})$. Let A be a uniformly distributed random number on $[0,1]$. Then the random subset $\omega \to S_{A(\omega)}(p_D)$ can be regarded as a *random FoV* that selects among a range of possible alternative cookie-cutter FoVs, whose shapes are specified by $p_D(\mathbf{x})$. This random set contains the same information as $p_D(\mathbf{x})$ since $p_D(\mathbf{x})$ can be recovered from it: $\Pr(\mathbf{x} \in S_a(p_D)) = \Pr(A \le p_D(\mathbf{x})) = p_D(\mathbf{x})$. Also, note that $E[\mathbf{1}_{S_A(p_D)}(\mathbf{x})] = p_D(\mathbf{x})$ where "$E[-]$" is expected value.

5.2 The hedged posterior p.g.fl.

Assume that the posterior p.g.fl. can be approximated as

$$G_{k+1|k+1}[h] = G[h] \cdot \prod\nolimits_{\mathbf{z} \in Z_{k+1}} \gamma_{\mathbf{z}}[h] \tag{29}$$

for some $G[h]$ such that $G[1] = 1$ and which has no dependence upon Z_{k+1}; and for some family of functionals $\gamma_{\mathbf{z}}[h]$ such that $\gamma_{\mathbf{z}}[h] = 1$ for all $\mathbf{z}$. (This will prove to be the case if Poisson-type approximations like that of Eq. (19) of section 3.4 are made.) Taking the logarithm,

$$\log G_{k+1|k+1}[h] = \log G[h] + \sum\nolimits_{\mathbf{z} \in Z_{k+1}} \log \gamma_{\mathbf{z}}[h].$$

Choose some fixed instantiation $S_a(p_D)$ of the random FoV $S_A(p_D)$. Then the log-posterior p.g.fl. must be

$$\log G_{k+1|k+1}[h] = \log G[h] + \sum_{i=1}^{\hat{n}} \mathbf{1}_{S_a(p_D)}(\hat{\mathbf{x}}_i) \cdot \log \gamma_{\eta(\hat{\mathbf{x}}_i)}[h]$$

Since this equation corresponds to only one possible FoV, we must produce an equation that corresponds to an "average FoV." So take the expectation of both sides:

$$\log \dot{G}_{k+1|k+1}[h] = \log G[h] + \sum_{i=1}^{\hat{n}} p_D(\mathbf{x}) \cdot \log \gamma_{\eta(\hat{\mathbf{x}}_i)}[h]$$

Taking the exponential we get the *hedged posterior p.g.fl.*:

$$\dot{G}_{k+1|k+1}[h] = G[h] \cdot \prod_{i=1}^{\hat{n}} \gamma_{\eta(\hat{\mathbf{x}}_i)}[h]^{p_D(\hat{\mathbf{x}}_i)} \tag{30}$$

5.3 PENT (single-sensor case)

According to Eq. (7), the PHD of the hedged posterior p.g.fl. may be computed as

$$\dot{D}_{k+1|k+1}(\mathbf{x}) = \frac{\delta \log \dot{G}_{k+1|k+1}}{\delta \mathbf{x}}[1] \tag{31}$$

and according to the formula just before Eq. (7), the posterior expected number of targets is:

$$\dot{N}_{k+1|k+1}(\mathbf{x}^*_{k+1}) = \int \dot{D}_{k+1|k+1}(\mathbf{x}) d\mathbf{x} \tag{32}$$

5.4 PENT (multisensor case)

For the sake of clarity assume two sensors with FoVs

$$p^1_D(\mathbf{x}) = p^1_D(\mathbf{x}, \mathbf{x}^{*1}_{k+1}), \quad p^2_D(\mathbf{x}) = p^2_D(\mathbf{x}, \mathbf{x}^{*2}_{k+1}).$$

and likelihood functions

$$L^1_{\mathbf{z}^1}(\mathbf{x}) = f^1_{k+1}(\mathbf{z}^1 | \mathbf{x}, \mathbf{x}^{*1}_{k+1}), \quad L^2_{\mathbf{z}^2}(\mathbf{x}) = f^2_{k+1}(\mathbf{z}^2 | \mathbf{x}, \mathbf{x}^{*2}_{k+1})$$

and Poisson false alarm models

$$\lambda^1 = \lambda^1_{k+1}, \qquad c^1(\mathbf{z}^1) = c^1_{k+1}(\mathbf{z}^1)$$

$$\lambda^2 = \lambda^2_{k+1}, \qquad c^2(\mathbf{z}^2) = c^2_{k+1}(\mathbf{z}^2)$$

Begin by modeling the two sensors as a single imaginary "pseudo-sensor." This allows us to apply the reasoning for the single-sensor case. (The discussion that follows is simplified. See [14] for a full treatment.) Since an ideal-observation set contains ideal observations collected from each target by at least one (but not necessarily both) sensors, we can take the probability of detection for the pseudo-sensor to be the joint multisensor FoV

$$\breve{p}_D(\mathbf{x}) = 1 - (1 - p^1_D(\mathbf{x})) \cdot (1 - p^2_D(\mathbf{x})) \tag{33}$$

where we abbreviate $\breve{p}_D(\mathbf{x}) = \breve{p}_D(\mathbf{x}, \mathbf{x}^{*1}_{k+1}, \mathbf{x}^{*2}_{k+1})$. This is the probability that at least one of the sensors will collect an observation if a target with state $\mathbf{x}$ is present. Assume that the pseudo-sensor collects observations of the form $\tilde{\mathbf{z}} = (\mathbf{z}^1, \mathbf{z}^2)$ and has the likelihood function

$$\tilde{L}_{(\mathbf{z}^1,\mathbf{z}^2)}(\mathbf{x}) = f^1_{k+1}(\mathbf{z}^1 | \mathbf{x}, \mathbf{x}^{*1}_{k+1}) \cdot f^2_{k+1}(\mathbf{z}^2 | \mathbf{x}, \mathbf{x}^{*2}_{k+1}). \tag{34}$$

Also, assume the following Poisson false alarm model:

$$\tilde{\lambda} = \lambda^1_{k+1} + \lambda^2_{k+1}, \qquad \tilde{c}(\mathbf{z}^1, \mathbf{z}^2) = c^1_{k+1}(\mathbf{z}^1) \cdot c^2_{k+1}(\mathbf{z}^2)$$

Then we assume that the approximation of Eq. (29) holds for the pseudo-sensor:

$$G_{k+1|k+1}[h] = G[h] \cdot \prod\nolimits_{(\mathbf{z}^1,\mathbf{z}^2) \in \tilde{Z}_{k+1}} \gamma_{(\mathbf{z}^1,\mathbf{z}^2)}[h] \tag{35}$$

Then the corresponding hedged p.g.fl. will be

$$\dot{G}_{k+1|k+1}[h] = G[h] \cdot \prod_{i=1}^{\hat{n}} \gamma_{(\eta^1(\hat{\mathbf{x}}_i), \eta^2(\hat{\mathbf{x}}_i))}[h]^{\breve{p}_D(\mathbf{x})} \tag{36}$$

and we compute PENT using Eqs. (31) and (32):

$$\dot{N}_{k+1|k+1}(\mathbf{x}^{*1}_{k+1}, \mathbf{x}^{*2}_{k+1}) = \int \frac{\delta \dot{G}_{k+1|k+1}}{\delta \mathbf{x}}[1] d\mathbf{x} \tag{37}$$

6 PENT (ideal sensor dynamics)

This section sketches the derivation of PENT for two cases: single-sensor, single-step look-ahead; and multisensor, single-step look-ahead. Similar formulas can be computed for multistep look-ahead (not described for lack of space). See [14] for a full discussion.

6.1 PENT (single-sensor, single-step)

In section 3.4 we noted that in the single-sensor, single-step look-ahead case the posterior p.g.fl. has the form

$$G_{k+1|k}[h] = G_{k+1}[h] \cdot \prod\nolimits_{\mathbf{z} \in Z_{k+1}} \gamma_{\mathbf{z}}[h] \tag{38}$$

where

$$\gamma_{\mathbf{z}}[h] = \frac{\lambda c(\mathbf{z}) + D_{k+1|k}[h p_D L_{\mathbf{z}}]}{\lambda c(\mathbf{z}) + D_{k+1|k}[p_D L_{\mathbf{z}}]} \tag{39}$$

$$G_{k+1}[h] = \exp\left(D_{k+1|k}[(h-1)(1-p_D)]\right) \tag{40}$$

This has the same form assumed in Eq. (29) for application of the maxi-PIMS optimization strategy. So we know that the hedged posterior p.g.fl. is

$$\dot{G}_{k+1|k+1}[h] = G[h] \cdot \prod_{i=1}^{\hat{n}} \gamma_{\eta(\hat{\mathbf{x}}_i)}[h]^{p_D(\hat{\mathbf{x}}_i)} \quad (41)$$

By Eq. (31) the corresponding PHD can be found by taking the first functional derivative of the log-p.g.fl.:

$$\dot{D}_{k+1|k+1}(\mathbf{x}) = (1 - p_D(\mathbf{x}))D_{k+1|k}(\mathbf{x}) + \sum_{i=1}^{\hat{n}} p_D(\hat{\mathbf{x}}_i) \cdot \frac{p_D(\mathbf{x})D_{k+1|k}(\mathbf{x})}{\lambda c(\eta(\hat{\mathbf{x}}_i)) + D_{k+1|k}[p_D L_{\eta(\hat{\mathbf{x}}_i)}]}$$

By Eq. (32) PENT is the integral of this:

$$N_{k+1|k+1}(\mathbf{x}^*_{k+1}) = D_{k+1|k}[1 - p_D] + \sum_{i=1}^{\hat{n}} p_D(\hat{\mathbf{x}}_i) \cdot \left(1 - \frac{\lambda c(\eta(\hat{\mathbf{x}}_i))}{\lambda c(\eta(\hat{\mathbf{x}}_i)) + D_{k+1|k}[p_D L_{\eta(\hat{\mathbf{x}}_i)}]}\right) \quad (42)$$

where we have applied partial. This formula is used with the PHD filter of section 3.3. It can be modified for use with the MHC filter of section 3.5 by using Eq. (24):

$$N_{k+1|k+1}(\mathbf{x}^*_{k+1}) = \sum_{j=1}^{N} q_j f_j[1 - p_D] + \sum_{j=1}^{N} p_D(\hat{\mathbf{x}}_j) \cdot \left(1 - \frac{\lambda c(\eta(\hat{\mathbf{x}}_j))}{\lambda c(\eta(\hat{\mathbf{x}}_j)) + \sum_{e=1}^{N} q_e f_e[p_D L_{\eta(\hat{\mathbf{x}}_j)}]}\right)$$

When there are no false alarms ($\lambda = 0$) this reduces to

$$N_{k+1|k+1}(\mathbf{x}^*_{k+1}) = \sum_{j=1}^{N} \left(q_j f_j[1 - p_D] + p_D(\hat{\mathbf{x}}_j)\right) \quad (43)$$

This formula can be computed in closed form if the FoV has the Gaussian form

$$p_D(\mathbf{x}) = \exp\left((A_{k+1}\mathbf{x} - A^*_{k+1}\mathbf{x}^*_{k+1})^T L^{-1} (A_{k+1}\mathbf{x} - A^*_{k+1}\mathbf{x}^*_{k+1})\right)$$

6.2 PENT (multisensor, single-step)

If we apply the pseudo-sensor approximation of section 5.4 then Eq. (42) can be directly applied to get a formula for PENT in the two-sensor, single-step look-ahead case:

$$N_{k+1|k+1}(\mathbf{x}^{*1}_{k+1}, \mathbf{x}^{*2}_{k+1}) = \sum_{j=1}^{N} q_j f_j[1 - \breve{p}_D] + \sum_{j=1}^{N} \breve{p}_D(\hat{\mathbf{x}}_j) \cdot \left(1 - \frac{\lambda c(\eta(\hat{\mathbf{x}}_j))}{\lambda c(\eta(\hat{\mathbf{x}}_j)) + \sum_{e=1}^{N} q_e f_e[\breve{p}_D L_{\eta^1(\hat{\mathbf{x}}_j)} L_{\eta^2(\hat{\mathbf{x}}_i)}]}\right)$$

When there are no false alarms this reduces to:

$$N_{k+1|k+1}(\mathbf{x}^{*1}_{k+1}, \mathbf{x}^{*2}_{k+1}) = \sum_{j=1}^{N} \left(q_j f_j[1 - \breve{p}_D] + \breve{p}_D(\hat{\mathbf{x}}_j)\right) \quad (44)$$

7 PENT with non-ideal sensor dynamics

We extend PENT to sensors such as those carried on UAVs, assumed of known constant number. Sensor motion is limited by physical or other constraints, and these motions are influenced indirectly by choosing control vectors $\mathbf{u}_k$ rather than directly by choosing future sensor states $\mathbf{x}^*_{k+1}$. Also, we assume that:

(1) each platform carries one sensor;

(2) for each sensor, each target generates at most one observation and no observation is generated by more than one target;

(3) each observation collected from a target is contaminated by the sensor noise process;

(4) for each sensor, any multitarget observation is contaminated by a Poisson false alarm process; and

(5) for each sensor, the state of that sensor is observed by an internal actuator sensor whose single observation may be contaminated by noise, registration error, etc.

In [14] we additionally assume that transmission of observations may be interrupted. We do not do so here because things simplify considerably otherwise.

We have three goals: derive the PHD predictor (section 7.2), corrector (section 7.3), and PENT (section 7.4). Mathematical preliminaries are discussed in section 7.1.

7.1 Preliminaries

We begin by generalizing the foundations described in section 2. When sensors are dynamic and sensor states cannot be controlled directly, we regard targets and sensors as a single stochastically evolving multi-object system. The joint state of the system will have the form

$$\breve{X} = \{\breve{\mathbf{x}}_1, \ldots, \breve{\mathbf{x}}_{\breve{n}}\}$$

where $\breve{n} = n + n^*$ with the number n^* of sensors known and fixed, the number n of targets random, and where each $\breve{\mathbf{x}}$ can be a target state or a sensor state: $\breve{\mathbf{x}} = \mathbf{x}$ or $\breve{\mathbf{x}} = \mathbf{x}^{*i}$. Functions $\breve{h}(\breve{\mathbf{x}})$ are defined on both target and sensor states: $\breve{h}(\mathbf{x})$ if $\breve{\mathbf{x}} = \mathbf{x}$ and $\breve{h}(\mathbf{x}^{*i})$ if $\breve{\mathbf{x}} = \mathbf{x}^{*i}$. In particular, PHDs are functions defined on the joint state: $\breve{D}(\mathbf{x})$ if $\breve{\mathbf{x}} = \mathbf{x}$ and $\breve{D}(\mathbf{x}^{*i})$ if $\breve{\mathbf{x}} = \mathbf{x}^{*i}$. So, predictor and corrector equations for such *joint PHDs* always have two parts: a predictor for targets, a predictor for sensors, a corrector for targets, and a corrector for sensors. Similarly, probability generating functionals $\breve{G}_{k|k}[\breve{h}]$ are defined on joint functions $\breve{h}(\breve{\mathbf{x}})$.

Observation-sets will have the form

$$\breve{Z} = \{\breve{\mathbf{z}}_1, \ldots, \breve{\mathbf{z}}_{\breve{m}}\}$$

where $\breve{m} = m + m^*$ with the number $m^* = n^*$ of actuator-sensor observations known and fixed, the number m of sensor observations is random, and $\breve{\mathbf{z}}$ is a sensor or actuator-sensor measurement: $\breve{\mathbf{z}} = \mathbf{z}$ or $\breve{\mathbf{z}} = \mathbf{z}^{*i}$.

Because we assume that actuator-sensor observations are perfectly detected, we may assume that the joint PHD on each sensor is actually a probability density:

$$\int \breve{D}_{k+1|k}(\mathbf{x}^{*i}) d\mathbf{x}^{*i} = 1 \quad \text{(all } i\text{)} \quad (45)$$

7.2 PHD predictor (single dynamic sensor)

· *Joint PHD predictor equation (targets):* Make the same motion assumptions stipulated in section 3.3. Then the predictor equation for the joint PHD $\breve{D}_{k+1|k}(\breve{\mathbf{x}})$ for target states $\breve{\mathbf{x}} = \mathbf{x}$ is the usual PHD predictor Eq. (16):

$$\breve{D}_{k+1|k}(\mathbf{x}) = b_{k+1|k}(\mathbf{x}) + \int s_{k+1|k}(\mathbf{w}) f_{k+1|k}(\mathbf{x} \mid \mathbf{w}) \breve{D}_{k|k}(\mathbf{w}) d\mathbf{w} \quad (46)$$

· *Joint PHD predictor equation (sensors):* Assume that the between-measurements motion of each sensor is described by a Markov transition $f^{*i}_{k+1|k}(\mathbf{y}^{*i} \mid \mathbf{x}^{*i})$. Then the predictor equation for the joint PHD for sensor states $\breve{\mathbf{x}} = \mathbf{x}^{*i}$ is an ordinary Bayes filter time-update:

$$\breve{D}_{k+1|k}(\mathbf{x}^{*i}) = \int f^{i}_{k+1|k}(\mathbf{x}^{*i} \mid \mathbf{w}^{*i}) \breve{D}_{k|k}(\mathbf{w}^{*i}) d\mathbf{w}^{*i} \qquad (47)$$

7.3 PHD corrector (single dynamic sensor)

The corrector equation for the joint PHD is as follows:

· *Joint PHD corrector equation (targets)*: Let probability of detection and likelihood for i^{th} sensor be

$$p^i_D(\mathbf{x}) = p_D(\mathbf{x}, \mathbf{x}^{*i}_{k+1}), \quad L^i_{\mathbf{z}^i}(\mathbf{x}) = f^i_{k+1}(\mathbf{z}^i \mid \mathbf{x}, \mathbf{x}^{*i}_{k+1})$$

respectively. Assume that the observations are corrupted by Poisson false alarm processes

$$\lambda^i = \lambda^i_{k+1}, \quad c^i(\mathbf{z}^i) = c^i_{k+1}(\mathbf{z}^i)$$

Then the corrector equation for the joint PHD $\breve{D}_{k+1|k+1}(\breve{\mathbf{x}})$ for target states $\breve{\mathbf{x}} = \mathbf{x}$ is:

$$\breve{D}_{k+1|k+1}(\mathbf{x}) = \breve{D}_{k+1|k}[1 - p^i_{D,\mathbf{x}}] \cdot \breve{D}_{k+1|k}(\mathbf{x}) + \sum_{\mathbf{z}^i \in Z^i_{k+1}}^{m} \frac{\breve{D}_{k+1|k}[p^i_{D,\mathbf{x}} L^i_{\mathbf{z}^i,\mathbf{x}}] \cdot \breve{D}_{k+1|k}(\mathbf{x})}{\lambda^i c^i(\mathbf{z}^i) + (\breve{D}_{k+1|k} \times \breve{D}_{k+1|k})[p^i_D L^i_{\mathbf{z}^i}]} \qquad (48)$$

Here we have abbreviated:

$$p^i_{D,\mathbf{x}}(\mathbf{x}^{*i}) = p_D(\mathbf{x}, \mathbf{x}^{*i}), \quad L^i_{\mathbf{z}^i,\mathbf{x}}(\mathbf{x}^{*i}) = f^i_{k+1}(\mathbf{z}^i \mid \mathbf{x}, \mathbf{x}^{*i})$$

$$(\breve{D} \times \breve{D})[\rho] = \int \breve{D}(\mathbf{x}) \cdot \breve{D}(\mathbf{x}^{*i}) \cdot \rho(\mathbf{x}, \mathbf{x}^{*i}) d\mathbf{x} d\mathbf{x}^{*i}$$

· *Joint PHD corrector equation (sensors)*: Assume that the state $\mathbf{x}^{*i}$ of the i'th sensor is observed by an internal actuator sensor, and that such observations $\mathbf{z}^{*i}$ are governed by a likelihood function

$$L^{*i}_{\mathbf{z}^{*i}}(\mathbf{x}^{*i}) = f^{*i}_{k+1}(\mathbf{z}^{*i} \mid \mathbf{x}^{*i})$$

Then the corrector for $\breve{D}_{k+1|k+1}(\breve{\mathbf{x}})$ for sensor states $\breve{\mathbf{x}} = \mathbf{x}^{*i}$ is an ordinary Bayes filter Bayes' rule data-update:

$$\breve{D}_{k+1|k+1}(\mathbf{x}^{*i}) = \frac{L^{*i}_{\mathbf{z}^i}(\mathbf{x}) \breve{D}_{k+1|k}(\mathbf{x}^{*i})}{D_{k+1|k}[L^{*i}_{\mathbf{z}^i}]} \qquad (49)$$

Eqs. (48) and (49) are derived as follows. As in section 3.4, everything hinges on finding a simple formula for $\breve{F}_{k+1}[\breve{g}.\breve{h}]$. Towards this end, the joint sensor-target observation model is linearized as follows:

· *Linearized actuator sensor p.g.fl.* : We assume that actuator sensor observations are governed by

$$\breve{F}^{act}_{k+1}[\breve{g}.\breve{h}] \cong \exp\left(-1 + \breve{D}_{k+1|k}[\breve{h} p^*_{\breve{g}}]\right)$$

where

$$p^*_{\breve{g}}(\mathbf{x}^*) = \int \breve{g}(\mathbf{z}^*) \cdot f^*_{k+1}(\mathbf{z}^* \mid \mathbf{x}^*) d\mathbf{z}^*$$

· *Linearized sensor p.g.fl.*: We assume that sensor observations are governed by

$$\breve{F}^{sens}_{k+1}[\breve{g}.\breve{h}] \cong \exp\left(-N_{k+1|k} + (\breve{D}_{k+1|k} \times \breve{D}_{k+1|k})[(\breve{h} \times 1)(1 - p_D + p_D p_{\breve{g}})]\right)$$

where

$$c_{\breve{g}} = \int \breve{g}(\mathbf{z}) \cdot c_{k+1}(\mathbf{z}) d\mathbf{z}$$

· *Sensor false alarm p.g.fl.*: Sensor observations are corrupted by a Poisson false alarm process of the form:

$$\breve{F}^{clutt}_{k+1}[\breve{g}] \cong \exp\left(-\lambda + \lambda c_{\breve{g}}\right)$$

· *Total linearized sensor p.g.fl.*: If the previous three models are conditionally independent then:

$$\breve{F}_{k+1}[\breve{g}.\breve{h}] \cong \exp\begin{pmatrix} -1 - \lambda - N_{k+1|k} + \lambda c_{\breve{g}} + \breve{D}_{k+1|k}[\breve{h} p^*_{\breve{g}}] \\ + (\breve{D}_{k+1|k} \times D_{k+1|k})[(\breve{h} \times 1)(1 - p_D + p_D p_{\breve{g}})] \end{pmatrix}$$

From Eq. (15) we get the following posterior p.g.fl.:

$$\breve{G}_{k+1|k}[\breve{h}] = \breve{G}_{k+1}[\breve{h}] \cdot \gamma_{\mathbf{z}^*_{k+1}}[\breve{h}] \cdot \prod\nolimits_{\mathbf{z} \in Z_{k+1}} \gamma_{\mathbf{z}}[\breve{h}] \qquad (50)$$

where

$$\gamma_{\mathbf{z}}[\breve{h}] = \frac{\lambda c(\mathbf{z}) + (\breve{D}_{k+1|k} \times \breve{D}_{k+1|k})[(\breve{h} \times 1) p_D L_{\mathbf{z}}]}{\lambda c(\mathbf{z}) + (\breve{D}_{k+1|k} \times \breve{D}_{k+1|k})[p_D L_{\mathbf{z}}]} \qquad (51)$$

$$\gamma_{\mathbf{z}^*}[\breve{h}] = \frac{\breve{D}_{k+1|k}[\breve{h} L_{\mathbf{z}^*}]}{\breve{D}_{k+1|k}[L_{\mathbf{z}^*}]} \qquad (52)$$

$$\breve{G}_{k+1}[\breve{h}] = \exp\left((\breve{D}_{k+1|k} \times \breve{D}_{k+1|k})[(\breve{h} \times 1 - 1)(1 - p_D)]\right) \qquad (53)$$

If we follow the procedure outlined in section 3.4 we get the claimed Eqs. (48) and (49).

7.4 PENT (single dynamic sensor)

The posterior p.g.fl. of Eq. (50) has the form assumed in Eq. (29) for application of the maxi-PIMS optimization strategy. Let $\hat{\mathbf{x}}^*_0$ be the predicted sensor state and assume that the actuator sensor likelihood has the additive form

$$L^*_{\mathbf{z}^*}(\mathbf{x}^*) = f^*_{k+1}(\mathbf{z}^* \mid \mathbf{x}^*) = f_{\mathbf{W}^*_{k+1}}(\mathbf{z}^* - \eta^*_{k+1}(\mathbf{x}^*))$$

where we abbreviate $\eta^*(\mathbf{x}^*) = \eta^*_{k+1}(\mathbf{x}^*)$. Then from Eq. (50) the hedged posterior p.g.fl. is

$$\dot{G}_{k+1|k+1}[\breve{h}] = G[\breve{h}] \cdot \gamma_{\eta^*(\mathbf{x}^*_0)} \cdot \prod_{i=1}^{\hat{n}} \gamma_{\eta(\hat{\mathbf{x}}_i)}[\breve{h}]^{p_D(\hat{\mathbf{x}}_i)} \qquad (54)$$

If we follow the procedure of section 5 we end up with the following formula for the PENT:

$$\dot{N}_{k+1|k+1}(\mathbf{u}_k) = (\breve{D}_{k+1|k} \times \breve{D}_{k+1|k})[1 - p_D] + \sum_{i=1}^{\hat{n}} p_D(\hat{\mathbf{x}}_i, \hat{\mathbf{x}}^*_0) \cdot \left(1 - \frac{\lambda c(\eta(\hat{\mathbf{x}}_i))}{\lambda c(\eta(\hat{\mathbf{x}}_i)) + (D_{k+1|k} \times D_{k+1|k})[p_D L_{\eta(\hat{\mathbf{x}}_i)}]}\right)$$

If there are no false alarms and if this equation is modified for use with the MHC filter of section 3.5 then

$$\dot{N}_{k+1|k+1}(\mathbf{u}_k) = \sum_{j=1}^{N} \left(q_j (f_j \times f^*_0)[1 - p_D] + p_D(\hat{\mathbf{x}}_i, \hat{\mathbf{x}}_0)\right) \qquad (55)$$

where $f^*_0(\mathbf{x}^*) = N_{P^*_0}(\mathbf{x}^* - \mathbf{x}^*_0)$ denotes the state distribution of the sensor and where $\mathbf{u}_k$ is the control-vector at time-step k.

7.5 PENT (multiple dynamic sensors)

The pseudo-sensor approximation described in section 5.4 can be applied to the dynamic-sensor case in much the same way as in the non-dynamic case described in section 6.2 [14]. For lack of space, we do not explore this further.

8 Conclusions

In this paper we have described a general system-level, control-theoretic approach for the management of mobile sensors whose states are indirectly observed by internal actuator sensors, but assuming no communication interference. Preliminary two-sensor, single-step look-ahead simulations using PENT with an MHC filter have demonstrated good sensor management behavior [3].

Ultimate proof of the utility of the approach depends, of course, upon continued demonstration of good behavior under increasingly realistic conditions. Even so, genuine progress depends on investigation of the deep stochastic structure of multisensor, multitarget sensor management problems. In [7] we argued that deeper insight is required if the inherent—but less than candidly acknowledged—computational "logjams" of the "plain-vanilla Bayesians" are to be surmounted. Indeed, the "plain-vanilla Bayesian approach" is an obstacle to such insight because of its obscurantist insistence that modeling and computational implementation must be "straightforward" (i.e., simplistically intertwined) and that anything else is "obfuscated." Deeper investigation is precisely what FISST dares to attempt.

This having been stated, our work still has significant limitations. We must assume that the sensors are fixed in number. This precludes the possibility of sensors entering or leaving a scenario. We must assume that each platform carries exactly one sensor. Our basic scheme is still centralized. Future work must address such issues.

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