Sequential Nonlinear Tracking Filter with Range-rate Measurements in Spherical Coordinates

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Abstract - *The three-dimensional CMKF-D with only position measurements is extended to solve the radar target tracking problem with range-rate measurements, where the errors between range and range-rate measurements are correlated. Firstly, a pseudo measurement is constructed by the product of range and range-rate measurements to reduce the high nonlinearity of the range-rate measurements with respect to the target state; then the consistent first two moment estimates of the converted measurement errors are derived by the nested conditioning method; finally, since the pseudo measurement is quadratic in the target state, the second-order EKF is used to implement the nonlinear tracking filtering, where the Cholesky factorization is used to decorrelate the converted position and pseudo measurement errors, thus the position and pseudo measurements can be sequentially processed to reduce the approximation error in the second-order EKF further. Monte-Carlo simulation results show that performance of the new tracking filter is improved greatly.*

Keywords: range-rate measurements, radar target tracking, CMKF-D, second-order EKF, sequential processing.

1 Introduction[1]

In tracking applications, target dynamics is usually modeled in Cartesian coordinates, while the measurements are directly available in the original sensor coordinates, most often in spherical or polar coordinates. Thus tracking in Cartesian coordinates using sensor coordinates measurements is actually a nonlinear estimation problem. For this, a basic idea is to transform the spherical measurements into a pseudo-linear form in the Cartesian coordinates, and then do target tracking by Kalman filtering with the estimated bias and covariance of the errors of the converted measurements. This is called the converted measurements Kalman filtering (CMKF) method. This method has been widely investigated by different ways of obtaining the bias and covariance of the converted measurement errors (e.g., [1~6]), but only the position measurements are considered. For a practical radar, in particular a Doppler radar, the range-rate measurement is also available. As shown in [7], target tracking accuracy can been greatly improved by sufficient utilization of the range-rate measurements.

To solve the radar target tracking problem with range-rate measurements, the conventional method is the extended Kalman filter (EKF) [8,9], but due to the nonlinearity between the radar measurements and the target state, performance of the tracking filter is often very poor. In [7], the product of the range and range-rate measurements is used to construct a pseudo measurement to reduce the nonlinearity of the range-rate measurement with respect to the target state , then statistics of the converted measurement errors of this pseudo measurement are obtained by the linearization method. [10] modifies the method proposed in [7] further, statistics of the position converted measurement errors are obtained by what is referred in [11] to as the nested conditioning method as in [2], but the pseudo measurement is processed still the same as in [7] by the linearization method. It has been proved in [2] that statistics of the converted measurement errors obtained by the linearization method are not consistent estimates. A sequential extended Kalman filter (SEKF) was proposed in [12] to process position and range-rate measurements sequentially. Due to the strong nonlinearity between the range-rate measurement and the target state, the range-rate measurements were processed by the EKF.

In addition, the range, angles and range-rate measurement errors are usually assumed statistically independent in existing radar target tracking algorithms, but recent research [13] shows that range and range-rate measurement errors are statistically correlated for some waveforms.

To use the range-rate measurement more sufficiently, the three-dimensional debiased consistent converted measurements Kalman filter (CMKF-D) with only position measurements in [2] is extended to solve the radar target tracking problem with range-rate measurements in this paper, where the errors between range and range-rate measurements are correlated. Performance improvement of the new tracking filter is verified by means of Monte-Carlo simulations.

The rest of this paper is organized as follows. Section 2 presents the target dynamic model and radar measurement equation. Section 3 derives the statistics of the converted

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measurement errors introduced by the range-rate measurements. Section 4 presents the tracking filter with the above converted measurements. Section 5 and 6 provide Monte-Carlo simulation results and conclusions, respectively.

2 Problem Formulation

2.1 Target Dynamic Model

The target dynamic model can be modeled in Cartesian coordinates as

$$X_{k+1} = \Phi_k X_k + G_k U_k + \Gamma_k W_k \tag{1}$$

where $X_k = [x_k \quad y_k \quad z_k \quad \dot{x}_k \quad \dot{y}_k \quad \dot{z}_k \quad s_{1\times(n-6)}]^T$ is the target motion state at time k, x_k, y_k, z_k are the position components along x, y, z directions, $\dot{x}_k, \dot{y}_k, \dot{z}_k$ are the corresponding velocity components, $s_{1\times(n-6)}$ are the other state components, e.g., acceleration and jerk components; $\Phi_k \in \boldsymbol{R}^{n\times n}$ is the state transition matrix; G_k, Γ_k are the coefficient matrices with appropriate dimensions; U_k is the deterministic input matrix; W_k is zero-mean white Gaussian process noise with known covariance Q_k.

2.2 Radar Measurement Equation

Assuming a three-coordinated radar located at the origin of the spherical coordinates, the radar measurement equation with range-rate measurements can be expressed as

$$Z_k^m = f_k(X_k) + V_k^m \tag{2}$$

where

$$Z_k^m = [r_k^m \quad \beta_k^m \quad \theta_k^m \quad \dot{r}_k^m]^T,$$

$$f_k(X_k) = [r_k \quad \beta_k \quad \theta_k \quad \dot{r}_k]^T,$$

$$r_k = \sqrt{x_k^2 + y_k^2 + z_k^2},$$

$$\beta_k = \tan^{-1}(z_k \big/ \sqrt{x_k^2 + y_k^2}),$$

$$\theta_k = \tan^{-1}(y_k / x_k),$$

$$\dot{r}_k = (x_k\dot{x}_k + y_k\dot{y}_k + z_k\dot{z}_k) \big/ \sqrt{x_k^2 + y_k^2 + z_k^2},$$

$$V_k^m = [\tilde{r}_k \quad \tilde{\beta}_k \quad \tilde{\theta}_k \quad \tilde{\dot{r}}_k]^T,$$

$r_k^m, \beta_k^m, \theta_k^m$ and $\dot{r}_k^m$ are radar measurements of the true target range r_k, elevation β_k, bearing θ_k and range-rate $\dot{r}_k$, respectively; $\tilde{r}_k, \tilde{\beta}_k, \tilde{\theta}_k$ and $\tilde{\dot{r}}_k$ are the corresponding measurement noises, which are all assumed to be zero-mean white Gaussian noises with known variances $\sigma_r^2, \sigma_\beta^2, \sigma_\theta^2$ and $\sigma_{\dot{r}}^2$, respectively; furthermore, it is assumed that $\tilde{r}_k, \tilde{\beta}_k$ and $\tilde{\theta}_k$ are statistically independent, $\tilde{\dot{r}}_k$ and $\tilde{\beta}_k, \tilde{\theta}_k$ are statistically independent, and $\tilde{\dot{r}}_k$ and $\tilde{r}_k$ is correlated with correlation coefficient ρ.

3 Measurement Conversion

3.1 Measurement Conversion with Range-rate Measurement

3.1.1 Position Measurements Conversion Equation

The position measurements (range, elevation and bearing) in spherical coordinates can be transformed into the pseudo-linear form in the Cartesian coordinates by

$$x_k^c = r_k^m \cos\beta_k^m \cos\theta_k^m = x_k + \tilde{x}_k \tag{3}$$

$$y_k^c = r_k^m \cos\beta_k^m \sin\theta_k^m = y_k + \tilde{y}_k \tag{4}$$

$$z_k^c = r_k^m \sin\beta_k^m = z_k + \tilde{z}_k \tag{5}$$

where $\tilde{x}_k, \tilde{y}_k, \tilde{z}_k$ are the position conversion measurement errors along x, y, z directions in Cartesian coordinates, respectively.

3.1.2 Pseudo Measurement Conversion Equation

To reduce the strong nonlinearity between the range-rate measurement and the target state, as in [7], the following pseudo measurement conversion equation can be utilized

$$\eta_k^c = r_k^m \dot{r}_k^m = x_k\dot{x}_k + y_k\dot{y}_k + z_k\dot{z}_k + \tilde{\eta}_k \tag{6}$$

where $\tilde{\eta}_k$ is the error of the converted pseudo measurement η_k^c in the Cartesian coordinates.

3.1.3 Measurements Conversion Equation

From (3) to (6), conversion of the radar measurements from the spherical coordinates to the Cartesian coordinates can be compactly expressed as

$$Z_k^c = h_k(X_k) + V_k^c \tag{7}$$

where

$$Z_k^c = [x_k^c \quad y_k^c \quad z_k^c \quad \eta_k^c]^T,$$

$$h_k(X_k) = [x_k \quad y_k \quad z_k \quad x_k\dot{x}_k + y_k\dot{y}_k + z_k\dot{z}_k]^T,$$

$$V_k^c = [\tilde{x}_k \quad \tilde{y}_k \quad \tilde{z}_k \quad \tilde{\eta}_k]^T.$$

3.2 Statistics of Converted Measurement Errors

3.2.1 True Bias and Covariance

Under the assumptions in (2), it can be shown (see the Appendix) that

$$E[\tilde{r}_k^2 \tilde{\dot{r}}_k] = E[\tilde{r}_k \tilde{\dot{r}}_k^2] = 0 \quad (8)$$

$$E[\tilde{r}_k^2 \tilde{\dot{r}}_k^2] = (1+2\rho^2)\sigma_r^2\sigma_{\dot{r}}^2 \quad (9)$$

Using (8) and (9), one can derive the true bias and covariance of the converted measurement errors conditioned on the true position and range-rate of the target as

$$\begin{aligned} \mu_{k,t} &= E[V_k^c \mid r_k, \beta_k, \theta_k, \dot{r}_k] \\ &= [\mu_{k,t}^x \quad \mu_{k,t}^y \quad \mu_{k,t}^z \quad \mu_{k,t}^\eta]^T \end{aligned} \quad (10)$$

$$\begin{aligned} R_{k,t} &= \mathrm{cov}[V_k^c, V_k^c \mid r_k, \beta_k, \theta_k, \dot{r}_k] \\ &= \begin{bmatrix} R_{k,t}^{xx} & R_{k,t}^{xy} & R_{k,t}^{xz} & R_{k,t}^{x\eta} \\ R_{k,t}^{xy} & R_{k,t}^{yy} & R_{k,t}^{yz} & R_{k,t}^{y\eta} \\ R_{k,t}^{xz} & R_{k,t}^{yz} & R_{k,t}^{zz} & R_{k,t}^{z\eta} \\ R_{k,t}^{x\eta} & R_{k,t}^{y\eta} & R_{k,t}^{z\eta} & R_{k,t}^{\eta\eta} \end{bmatrix} \end{aligned} \quad (11)$$

where

$\mu_{k,t}^x, \mu_{k,t}^y, \mu_{k,t}^z$ and $R_{k,t}^{xx}, R_{k,t}^{xy} R_{k,t}^{xz}, R_{k,t}^{yy}, R_{k,t}^{yz}, R_{k,t}^{zz}$ are the same as in [2],

$\mu_{k,t}^\eta = \rho\sigma_r\sigma_{\dot{r}}$,

$R_{k,t}^{x\eta} = (\sigma_r^2 \dot{r}_k + r_k \rho\sigma_r\sigma_{\dot{r}})\cos\beta_k \cos\theta_k \cdot e^{-\sigma_\beta^2/2-\sigma_\theta^2/2}$,

$R_{k,t}^{y\eta} = (\sigma_r^2 \dot{r}_k + r_k \rho\sigma_r\sigma_{\dot{r}})\cos\beta_k \sin\theta_k \cdot e^{-\sigma_\beta^2/2-\sigma_\theta^2/2}$,

$R_{k,t}^{z\eta} = (\sigma_r^2 \dot{r}_k + r_k \rho\sigma_r\sigma_{\dot{r}})\sin\beta_k \cdot e^{-\sigma_\beta^2/2}$,

$R_{k,t}^{\eta\eta} = r_k^2\sigma_{\dot{r}}^2 + \sigma_r^2\dot{r}_k^2 + (1+\rho^2)\sigma_r^2\sigma_{\dot{r}}^2 + 2r_k\dot{r}_k\rho\sigma_r\sigma_{\dot{r}}$.

3.2.2 Average True Bias and Covariance

Since (10) and (11) are conditioned on the true position and range-rate of the target, which are not available in practice, they can not be used directly. To make them more practicable, the expected value of the true bias and covariance are evaluated conditioned on the measured position and range-rate.

$$\begin{aligned} \mu_{k,a} &= E[\mu_{k,t} \mid r_k^m, \beta_k^m, \theta_k^m, \dot{r}_k^m] \\ &= [\mu_{k,a}^x \quad \mu_{k,a}^y \quad \mu_{k,a}^z \quad \mu_{k,a}^\eta]^T \end{aligned} \quad (12)$$

$$\begin{aligned} R_{k,a} &= E[R_{k,t} \mid r_k^m, \beta_k^m, \theta_k^m, \dot{r}_k^m] \\ &= \begin{bmatrix} R_{k,a}^{xx} & R_{k,a}^{xy} & R_{k,a}^{xz} & R_{k,a}^{x\eta} \\ R_{k,a}^{xy} & R_{k,a}^{yy} & R_{k,a}^{yz} & R_{k,a}^{y\eta} \\ R_{k,a}^{xz} & R_{k,a}^{yz} & R_{k,a}^{zz} & R_{k,a}^{z\eta} \\ R_{k,a}^{x\eta} & R_{k,a}^{y\eta} & R_{k,a}^{z\eta} & R_{k,a}^{\eta\eta} \end{bmatrix} \end{aligned} \quad (13)$$

where

$\mu_{k,a}^x, \mu_{k,a}^y, \mu_{k,a}^z$ and $R_{k,a}^{xx}, R_{k,a}^{xy} R_{k,a}^{xz}, R_{k,a}^{yy}, R_{k,a}^{yz}, R_{k,a}^{zz}$ are the same as in [2],

$\mu_{k,a}^\eta = \rho\sigma_r\sigma_{\dot{r}}$,

$R_{k,a}^{x\eta} = (\sigma_r^2 \dot{r}_k^m + r_k^m \rho\sigma_r\sigma_{\dot{r}})\cos\beta_k^m \cos\theta_k^m e^{-\sigma_\beta^2-\sigma_\theta^2}$,

$R_{k,a}^{y\eta} = (\sigma_r^2 \dot{r}_k^m + r_k^m \rho\sigma_r\sigma_{\dot{r}})\cos\beta_k^m \sin\theta_k^m e^{-\sigma_\beta^2-\sigma_\theta^2}$,

$R_{k,a}^{z\eta} = (\sigma_r^2 \dot{r}_k^m + r_k^m \rho\sigma_r\sigma_{\dot{r}})\sin\beta_k^m e^{-\sigma_\beta^2}$,

$$\begin{aligned} R_{k,a}^{\eta\eta} = (r_k^m)^2\sigma_{\dot{r}}^2 + \sigma_r^2(\dot{r}_k^m)^2 + 3(1+\rho^2)\sigma_r^2\sigma_{\dot{r}}^2 + \\ 2r_k^m\dot{r}_k^m\rho\sigma_r\sigma_{\dot{r}} \end{aligned}.$$

$\mu_{k,a}$ and $R_{k,a}$ are called the average true bias and average true covariance, respectively in [1,2].

Generally speaking, due to the nonlinear transformation in (3)~(6), although the measurement noises in the spherical Coordinates have a Gaussian distribution, the converted measurement errors are not Gaussian distributed any more. But by using the analogous consistency test method in [1,2,3], one can verify that (12) and (13) are consistent estimates of the first two moments of the converted measurement errors. So in practical tracking, the converted measurement (7) can be used to substitute (2).

4 Tracking Filter

It can be seen from (7) that the converted measurements are still nonlinear function of the target state. The conventional tracking filter for this is the EKF, in which $h_k(X_k)$ is linearized by the Taylor series expansion around $\hat{X}_{k/k-1}$. However, the position converted measurements are linear functions of the target state. So if sequential filtering is taken, that is, the position conversion measurements are processed first to obtain the target state estimate $\hat{X}_{k/k}^p$, then the nonlinear function $h_k(X_k)$ is linearized by the Taylor series expansion around $\hat{X}_{k/k}^p$, and thus the linearization errors should be reduced.

4.1 Decorrelation between Position and Pseudo Measurement

From (13), the converted measurement errors of position and pseudo measurement η_k^c are correlated, so they should be decorrelated first before sequential filtering.

Covariance matrix $R_{k,a}$ of the converted measurement errors can be rewritten as

$$R_{k,a} = \begin{bmatrix} R_{k,a}^{pp} & (R_{k,a}^{\eta p})^T \\ R_{k,a}^{\eta p} & R_{k,a}^{\eta\eta} \end{bmatrix}$$

where

$$R_{k,a}^{pp} = \begin{bmatrix} R_{k,a}^{xx} & R_{k,a}^{xy} & R_{k,a}^{xz} \\ R_{k,a}^{xy} & R_{k,a}^{yy} & R_{k,a}^{yz} \\ R_{k,a}^{xz} & R_{k,a}^{yz} & R_{k,a}^{zz} \end{bmatrix},$$

$$R_{k,a}^{\eta p} = [R_{k,a}^{x\eta} \quad R_{k,a}^{y\eta} \quad R_{k,a}^{z\eta}],$$

Let $L_k = -R_{k,a}^{\eta p}(R_{k,a}^{pp})^{-1} = [L_k^1 \quad L_k^2 \quad L_k^3], B_k = \begin{bmatrix} I_3 & 0 \\ L_k & 1 \end{bmatrix}$.

Pre-multiplying B_k on both sides of (7), from Cholesky factorization [14], one can get

$$Z_k^{c,p} = H_k^{c,p} X_k + V_k^{c,p} \tag{14}$$

$$\varepsilon_k^c = h_k^\varepsilon(X_k) + \tilde{\varepsilon}_k \tag{15}$$

where

$Z_k^{c,p} = [x_k^c \quad y_k^c \quad z_k^c]^T$,

$H_k^{c,p} = [I_3 \quad 0_{3\times(n-3)}]$,

$V_k^{c,p} = [\tilde{x}_k \quad \tilde{y}_k \quad \tilde{z}_k]^T$ with mean $\mu_{k,a}^p = [\mu_{k,a}^x \quad \mu_{k,a}^y \quad \mu_{k,a}^z]^T$ and variance $R_{k,a}^{pp}$,

$\varepsilon_k^c = L_k^1 x_k^c + L_k^2 y_k^c + L_k^3 z_k^c + \eta_k^c$,

$$h_k^\varepsilon(X_k) = L_k^1 x_k + L_k^2 y_k + L_k^3 z_k + x_k\dot{x}_k + y_k\dot{y}_k + z_k\dot{z}_k,$$

$\tilde{\varepsilon}_k = L_k^1\tilde{x}_k + L_k^2\tilde{y}_k + L_k^3\tilde{z}_k + \tilde{\eta}_k$ with mean $\mu_{k,a}^\varepsilon = L_k^1\mu_{k,a}^x + L_k^2\mu_{k,a}^y + L_k^3\mu_{k,a}^z + \mu_{k,a}^\eta$ and variance $R_{k,a}^{\varepsilon\varepsilon} = R_{k,a}^{\eta\eta} - R_{k,a}^{\eta p}(R_{k,a}^{pp})^{-1}(R_{k,a}^{\eta p})^T$, and $\tilde{\varepsilon}_k$ is uncorrelated with $V_k^{c,p}$.

4.2 Sequential Filtering

4.2.1 Position Measurement Filtering

Time update and measurement update of the target state by position measurement $Z_k^{c,p}$ can be implemented as follows.

$$\hat{X}_{k/k-1} = \Phi_{k-1}\hat{X}_{k-1/k-1} + G_{k-1}U_{k-1}$$

$$P_{k/k-1} = \Phi_{k-1}P_{k-1/k-1}\Phi_{k-1}^T + \Gamma_{k-1}Q_{k-1}\Gamma_{k-1}^T$$

$$K_k^p = P_{k/k-1}(H_k^{c,p})^T[H_k^{c,p}P_{k/k-1}(H_k^{c,p})^T + R_{k,a}^{pp}]^{-1}$$

$$\hat{X}_{k/k}^p = \hat{X}_{k/k-1} + K_k^p[Z_k^{c,p} - \mu_{k,a}^p - H_k^{c,p}\hat{X}_{k/k-1}]$$

$$P_{k/k}^p = (I_n - K_k^p H_k^{c,p})P_{k/k-1}$$

4.2.2 Update with Pseudo Measurement

From (15), the pseudo measurement ε_k^c is a quadratic function of the target state, and so the nonlinear filtering estimation for the target state can be achieved by the second-order EKF in [7] as

$$K_k^\varepsilon = P_{k/k}^p(H_k^\varepsilon)^T[H_k^\varepsilon P_{k/k}^p(H_k^\varepsilon)^T + R_{k,a}^{\varepsilon\varepsilon} + A_k]^{-1}$$

$$\hat{X}_{k/k}^\varepsilon = \hat{X}_{k/k}^p + K_k^\varepsilon[\varepsilon_k^c - \mu_{k,a}^\varepsilon - h_k^\varepsilon(\hat{X}_{k/k}^p) - \frac{1}{2}\delta_k^2]$$

$$P_{k/k}^\varepsilon = (I_n - K_k^\varepsilon H_k^\varepsilon)P_{k/k}^p$$

where

H_k^ε is the Jacobian of $h_k^\varepsilon(X_k)$ around $\hat{X}_{k/k}^p$, and

$$H_k^\varepsilon = [L_k^1 + \hat{x}_{k/k}^p \quad L_k^2 + \hat{y}_{k/k}^p \quad L_k^3 + \hat{z}_{k/k}^p \quad \hat{x}_{k/k}^p \quad \hat{y}_{k/k}^p \quad \hat{z}_{k/k}^p \quad 0_{1\times(n-6)}],$$

δ_k^2 consists of the second-order derivative of $h_k^\varepsilon(X_k)$, and $\delta_k^2 = 2P_{k/k}^p(1,4) + 2P_{k/k}^p(2,5) + 2P_{k/k}^p(3,6)$,

$$\begin{aligned} A_k = {} & P_{k/k}^p(1,1)P_{k/k}^p(4,4) + P_{k/k}^p(2,2)P_{k/k}^p(5,5) + \\ & P_{k/k}^p(3,3)P_{k/k}^p(6,6) + 2P_{k/k}^p(1,2)P_{k/k}^p(4,5) + \\ & 2P_{k/k}^p(1,5)P_{k/k}^p(2,4) + 2P_{k/k}^p(1,3)P_{k/k}^p(4,6) + \\ & 2P_{k/k}^p(1,6)P_{k/k}^p(3,4) + 2P_{k/k}^p(2,3)P_{k/k}^p(5,6) + \\ & 2P_{k/k}^p(2,6)P_{k/k}^p(3,5) + [P_{k/k}^p(1,4)]^2 + \\ & [P_{k/k}^p(2,5)]^2 + [P_{k/k}^p(3,6)]^2 \end{aligned}$$

$P_{k/k}^p(i, j)$ represents the element located at the ith row and jth column of $P_{k/k}^p$.

4.2.3 Final Filtering

The final target state filtering estimate and corresponding error covariance at time k are then given by

$$\hat{X}_{k/k} = \hat{X}_{k/k}^\varepsilon \qquad P_{k/k} = P_{k/k}^\varepsilon$$

5 Simulation and Comparison

To test the performance of the new nonlinear tracking filter with range-rate measurements, a target with a nearly constant velocity motion in the plane is considered. The target dynamic model in (1) is

$$X_{k+1} = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} X_k + \begin{bmatrix} T^2/2 & 0 \\ 0 & T^2/2 \\ T & 0 \\ 0 & T \end{bmatrix} \begin{bmatrix} w_k^x \\ w_k^y \end{bmatrix}$$

where $X_k = [x_k \quad y_k \quad \dot{x}_k \quad \dot{y}_k]^T$, the sampling interval $T = 1s$, w_k^x and w_k^y are zero-mean uncorrelated Gaussian white process noises with known standard deviation $q_k^x = q_k^y = 0.05m/s^2$.

The target is initially located at (3km, 3km), with an initial velocity are (15m/s, 15m/s). A Doppler radar is located at the origin of the polar coordinate, which measures the tracker-to-target range, bearing and range-rate with standard measurement noise deviation $\sigma_r = 100m$, $\sigma_\theta = 14mrad$, $\sigma_{\dot{r}} = 0.1m/s$, respectively. The correlation coefficient between the range and range-rate measurement noises is $\rho = 0.5$.

Assuming the measurements are available at $k = -1, 0$, the initial estimate for the target state is obtained using the first two measurements in a manner similar to that of two-point differencing method given in [14].

$$\hat{X}_{0/0} = [x_0^c \quad y_0^c \quad (x_0^c - x_{-1}^c)/T \quad (y_0^c - y_{-1}^c)/T]^T$$

and the corresponding error covariance is

$$P_{0/0} = \begin{bmatrix} P_{0/0}^{pp} & P_{0/0}^{pv} \\ (P_{0/0}^{pv})^T & P_{0/0}^{vv} \end{bmatrix},$$

where

$$P_{0/0}^{pp} = \begin{bmatrix} R_{0,a}^{xx} & R_{0,a}^{xy} \\ R_{0,a}^{xy} & R_{0,a}^{yy} \end{bmatrix}, \; P_{0/0}^{pv} = \frac{1}{T}\begin{bmatrix} R_{0,a}^{xx} & R_{0,a}^{xy} \\ R_{0,a}^{xy} & R_{0,a}^{yy} \end{bmatrix},$$

$$P_{0/0}^{vv} = \begin{bmatrix} \frac{T^2(q_{-1}^x)^2}{4} + \frac{R_{-1,a}^{xx} + R_{0,a}^{xx}}{T^2} & \frac{R_{-1,a}^{xy} + R_{0,a}^{xy}}{T^2} \\ \frac{R_{-1,a}^{xy} + R_{0,a}^{xy}}{T^2} & \frac{T^2(q_{-1}^y)^2}{4} + \frac{R_{-1,a}^{yy} + R_{0,a}^{yy}}{T^2} \end{bmatrix}$$

For performance comparison, RMS position and velocity errors given by

$$RMS\,Pos_k = \sqrt{\frac{1}{M}\sum_{i=1}^{M}[(x_k^i - \hat{x}_{k/k}^i)^2 + (y_k^i - \hat{y}_{k/k}^i)^2]}$$

$$RMS\,Vel_k = \sqrt{\frac{1}{M}\sum_{i=1}^{M}[(\dot{x}_k^i - \hat{\dot{x}}_{k/k}^i)^2 + (\dot{y}_k^i - \hat{\dot{y}}_{k/k}^i)^2]}$$

are computed over 300 steps, where $M = 200$ is the Monte-Carlo simulation runs, x_k^i, y_k^i and $\dot{x}_k^i, \dot{y}_k^i$ stand for the true position and velocity along x, y directions in the plane, $\hat{x}_k^i, \hat{y}_k^i$ and $\hat{\dot{x}}_k^i, \hat{\dot{y}}_k^i$ are the corresponding estimates.

The RMS position and velocity errors are shown in Figs. 1 and 2 for the new algorithm of this paper (RCMKF-D), the CMKF-D with only position measurements in [1], the EKF which processes the radar position and range-rate measurements simultaneously, and the SEKF in [8].

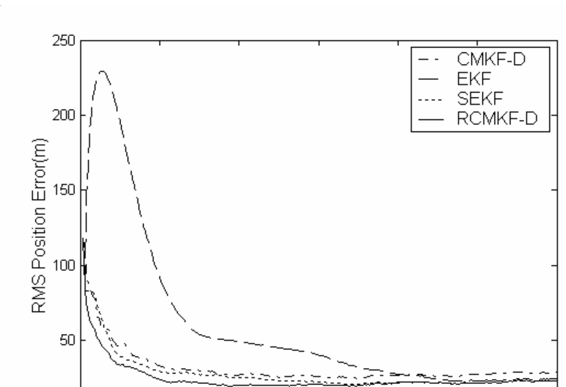

Fig. 1. RMS Position Error.

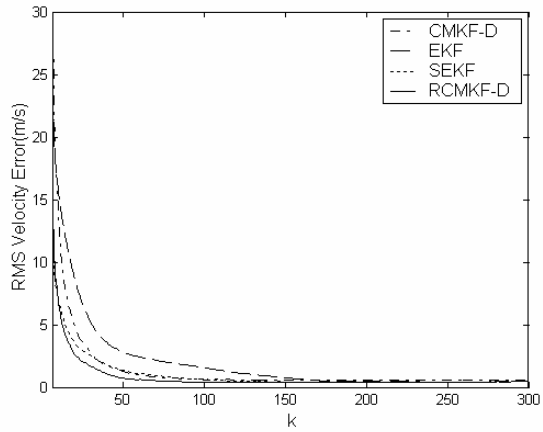

Fig. 2. RMS Velocity Error.

As can be seen, compared with the CMKF-D, the proposed tracking filter performs better both in the transient and steady-state errors. This further verifies that incorporation of range-rate measurements can improve performance of the radar tracking greatly. The EKF has very large transient errors and long convergence time, but the two algorithms asymptotically coincide with each other. The proposed filter also performs better than the SEKF, especially during transients.

6 Conclusions

Based on the three-dimensional debiased consistent converted measurement Kalman filter with only position measurements, a new nonlinear tracking filter with range-rate measurements, where the errors between range and range-rate measurements are correlated, is presented in this paper. Compared with the existing algorithms, it has the following advantages: a) product of range and range-rate measurement is utilized to construct an pseudo measurement, and thus the nonlinearity between the range-rate measurement and the target state can be reduced; b) the consistent first two moment estimates of the converted measurement errors are explicitly derived by the nested conditioning method without any linearization approximation; c) the pseudo measurement is exactly a quadratic function of the target state, and so a second-order EKF can solve the nonlinear tracking problem; d) the position and pseudo measurements are sequentially processed to improve the accuracy of the second-order EKF further. Monte-Carlo simulation results show that the proposed tracking filter outperforms the other alternatives.

Appendix

Equations (8) and (9) can be shown as follows.

From the joint Gaussian assumptions in (2), $\tilde{r}_k$ and $\tilde{\dot{r}}_k$ have the following joint probability density function:

$$f(s,t)=\frac{1}{2\pi\sigma_r\sigma_{\dot r}\sqrt{1-\rho^2}}\exp[\frac{-1}{2(1-\rho^2)} \\ (\frac{s^2}{\sigma_r^2}-2\rho\frac{st}{\sigma_r\sigma_{\dot r}}+\frac{t^2}{\sigma_{\dot r}^2})] \tag{16}$$

So the first third-order moment in (8) can be obtained as

$$E[\tilde{r}_k^2\tilde{\dot r}_k]=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}s^2t\cdot f(s,t)dsdt \tag{17}$$

In (16),

$$\frac{s^2}{\sigma_r^2}-2\rho\frac{st}{\sigma_r\sigma_{\dot r}}+\frac{t^2}{\sigma_{\dot r}^2} \\ =(\frac{s}{\sigma_r}-\frac{\rho t}{\sigma_{\dot r}})^2+(\sqrt{1-\rho^2}\,\frac{t}{\sigma_{\dot r}})^2 \tag{18}$$

Define

$$u=\frac{1}{\sqrt{1-\rho^2}}\left(\frac{s}{\sigma_r}-\frac{\rho t}{\sigma_{\dot r}}\right) \tag{19}$$

$$v=\frac{t}{\sigma_{\dot r}} \tag{20}$$

Then (17) can be rewritten as

$$E[\tilde{r}_k^2\tilde{\dot r}_k]=\frac{\sigma_r^2\sigma_{\dot r}}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(\sqrt{1-\rho^2}u+\rho v)^2 \\ v\cdot\exp(-\frac{u^2+v^2}{2})dudv \tag{21}$$

where

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}u^2v\exp(-\frac{u^2+v^2}{2})dudv \\ =\int_{-\infty}^{+\infty}u^2e^{-\frac{u^2}{2}}du\int_{-\infty}^{+\infty}ve^{-\frac{v^2}{2}}dv \\ =0 \tag{22}$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}uv^2\exp(-\frac{u^2+v^2}{2})dudv \\ =\int_{-\infty}^{+\infty}ue^{-\frac{u^2}{2}}du\int_{-\infty}^{+\infty}v^2e^{-\frac{v^2}{2}}dv \\ =0 \tag{23}$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}v^3\exp(-\frac{u^2+v^2}{2})dudv \\ =\int_{-\infty}^{+\infty}e^{-\frac{u^2}{2}}du\int_{-\infty}^{+\infty}v^3e^{-\frac{v^2}{2}}dv \\ =0 \tag{24}$$

Substituting (22)-(24) into (21) yields $E[\tilde{r}_k^2\tilde{\dot r}_k]=0$.

The second third-second moment $E[\tilde{r}_k\tilde{\dot r}_k^2]=0$ in (8) can be obtained similarly.

Similarly as the derivations of (8), (9) can be obtained as

$$E[\tilde{r}_k^2\tilde{\dot r}_k^2]=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}s^2t^2\cdot f(s,t)dsdt \\ =\frac{\sigma_r^2\sigma_{\dot r}^2}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(\sqrt{1-\rho^2}u+\rho v)^2 \\ v^2\cdot\exp(-\frac{u^2+v^2}{2})dudv \tag{25}$$

where

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}u^2v^2\exp(-\frac{u^2+v^2}{2})dudv \\ =\int_{-\infty}^{+\infty}u^2e^{-\frac{u^2}{2}}du\int_{-\infty}^{+\infty}v^2e^{-\frac{v^2}{2}}dv \\ =2\pi \tag{26}$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}uv^3\exp(-\frac{u^2+v^2}{2})dudv \\ =\int_{-\infty}^{+\infty}ue^{-\frac{u^2}{2}}du\int_{-\infty}^{+\infty}v^3e^{-\frac{v^2}{2}}dv \\ =0 \tag{27}$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}v^4\exp(-\frac{u^2+v^2}{2})dudv \\ =\int_{-\infty}^{+\infty}e^{-\frac{u^2}{2}}du\int_{-\infty}^{+\infty}v^4e^{-\frac{v^2}{2}}dv \\ =2\pi\cdot 3 \tag{28}$$

Substituting (26)-(28) into (25) yields $E[\tilde{r}_k^2\tilde{\dot r}_k^2]=(1+2\rho^2)\sigma_r^2\sigma_{\dot r}^2$, thus (9) also holds.

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