Target Tracking with Generalized Data Association based on the General DSm Rule of Combination*

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Abstract – *The objective of this paper is to present an approach for target tracking, which incorporates the advanced concept of generalized data (kinematics and attribute) association to improve track maintenance performance in complicated situations (closely spaced targets), when kinematics data are insufficient for correct decision making. It uses Global Nearest Neighbour-like approach and Munkres algorithm to resolve the generalized association matrix. The main peculiarity consists in applying the principles of Dezert-Smarandache theory of plausible and paradoxical reasoning to model and process the utilized attribute data. The new general Dezert-Smarandache hybrid rule of combination is used to deal with particular integrity constraints associated with some elements of the free Dedekind's distributive lattice. The aim of the performed study is to provide coherent decision making process related to generalized data association and to improve the overall tracking performance.*

Keywords: Target Tracking, Generalized Data Association, Dezert-Smarandache Theory, DSm hybrid rule of combination.

1 Introduction

One important function of each radar surveillance system is to keep and improve targets tracks maintenance performance. It becomes a crucial and challenging problem especially in complicated situations of closely spaced, or crossing targets. The design of a modern multitarget tracking (MTT) algorithms in a such real-life stressful environment motivates the incorporation of the advanced concepts for generalized data association. In order to resolve correlation ambiguities and to select the best observation-track pairings, in this paper, a particular generalized data association approach is proposed and incorporated in a MTT algorithm. It allows the introduction of target attribute into the association logic, based on the general Dezert-Smarandache rule for combination, which is adapted to deal with possible integrity constraints on the problem under consideration due to the true nature of the elements involved into it.

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2 Dezert-Smarandache Theory

The new Dezert-Smarandache Theory of plausible, uncertain and paradoxical reasoning (DSmT) [10, 11, 12, 16] proposes a new general and mathematical framework for solving fusion problems. This theory overcomes the practical limitations of the Dempster-Shafer Theory, coming essentially from its inherent constraints, which are closely related with the acceptance of the third exclude principle.

The foundations of the DSmT is to refute the principle of the third exclude and to allow imprecise/vague notions and concepts between elements of the frame of discernment Θ. The DSmT includes the possibility to deal with evidences arising from different sources of information which don't have access to absolute interpretation of the elements of Θ under consideration and can be interpreted as a general and direct extension of probability theory and the DST.

2.1 Free-DSm model

Let $\Theta = \{\theta_1, \dots, \theta_n\}$ be a set of n elements which cannot be precisely defined and separated. A free-DSm model, denoted as $\mathcal{M}^f(\Theta)$, consists in assuming that all elements θ_i, $i = 1, \dots, n$ of Θ are not exclusive [12]. The free-DSm model is an opposite to the Shafer's model $\mathcal{M}^0(\Theta)$, which requires the exclusivity and exhaustivity of all elements θ_i, $i = 1, \dots, n$ of Θ.

2.2 Hyper-power Set and Classical DSm Rule of Combination

The hyper-power set D^Θ is defined as the set of all composite possibilities build from Θ with $\cup$ and $\cap$ operators such that:

1. $\emptyset, \theta_1, \dots, \theta_n \in D^\Theta$
2. $\forall A \in D^\Theta, B \in D^\Theta, (A \cup B) \in D^\Theta, (A \cap B) \in D^\Theta$

3. No other elements belong to D^Θ , except those, obtained by using rules 1 or 2.

The cardinality of D^Θ is majored by 2^{2^n} when $\text{card}(\Theta) = |\Theta| = n$. The generation of hyper-power set D^Θ is closely related with the famous Dedekind's problem on enumerating the set of monotone Boolean functions.

From a general frame of discernement Θ with its free-DSm model, it is defined a map $m(.) : D^\Theta \to [0,1]$, associated to a given source of evidence $\mathcal{B}$, which can support paradoxical (or intrinsic conflicting) information, as follows :

$$m(\emptyset) = 0 \qquad \text{and} \qquad \sum_{A \in D^\Theta} m(A) = 1$$

The quantity $m(A)$ is called A's *general basic belief number* (gbba) or the general basic belief mass for A. The belief and plausibility functions are defined in almost the same manner as within the DST (Dempster-Shafer Theory), i.e.

$$\text{Bel}(A) = \sum_{B \in D^\Theta, B \subseteq A} m(B)$$

$$\text{Pl}(A) = \sum_{B \in D^\Theta, B \cap A \neq \emptyset} m(B)$$

and $\forall A \in D^\Theta, \text{Bel}(A) \leq \text{Pl}(A)$.

The classical DSm rule of combination of intrinsic conflicting and/or uncertain sources of information is based on the free-DSm model. For $k \geq 2$ independent bodies of evidence with general information granules $m_1(.)$, $\ldots$, $m_k(.)$ over D^Θ, the classical DSm rule of combination $m(.) \triangleq [m_1 \oplus \ldots \oplus m_k](.)$ is given by [10, 11] : $\forall A \neq \emptyset \in D^\Theta$,

$$m_{\mathcal{M}^f(\Theta)}(A) = \sum_{\substack{X_1,\ldots,X_k \in D^\Theta \\ X_1 \cap \ldots \cap X_k = A}} \prod_{i=1}^{k} m_i(X_i) \qquad (1)$$

and with $m_{\mathcal{M}^f(\Theta)}(\emptyset) = 0$ by definition. This rule, dealing with uncertain and/or paradoxical/conflicting information is commutative and associative and requires no normalization procedure.

2.3 Definition of a DSm hybrid Model

A DSm hybrid model is defined from the free-DSm model $\mathcal{M}^f(\Theta)$ by introducing some integrity constraints on some elements $A \in D^\Theta$, if there are some certain facts in accordance with the exact nature of the model related to the problem under consideration [14]. An integrity constraint on $A \in D^\Theta$ consists in forcing A to be empty through the model $\mathcal{M}$, denoted as $A \overset{\mathcal{M}}{\equiv} \emptyset$.

There are several possible kinds of integrity constraints introduced in any free-DSm model:

- *Exclusivity constraints*: when some conjunctions of elements of Θ are truly impossible, by example when $\theta_i \cap \ldots \cap \theta_k \overset{\mathcal{M}}{\equiv} \emptyset$.
- *Non-existential constraints*: when some disjunctions of elements of Θ are truly impossible, by example when $\theta_i \cup \ldots \cup \theta_k \overset{\mathcal{M}}{\equiv} \emptyset$. The vacuous DSm hybrid model $\mathcal{M}_\emptyset$, defined by constraint according to the total ignorance: $I_t \triangleq \theta_1 \cup \theta_2 \cup \ldots \cup \theta_n \overset{\mathcal{M}}{\equiv} \emptyset$, is excluded from consideration, because it is meaningless.
- *Mixture of exclusivity and non-existential constraints*: like for example $(\theta_i \cap \theta_j) \cup \theta_k$ or any other hybrid proposition/element of D^Θ involving both $\cap$ and $\cup$ operators such that at least one element θ_k is subset of the constrained proposition.

The introduction of a given integrity constraint $A \overset{\mathcal{M}}{\equiv} \emptyset \in D^\Theta$ implies the set of inner constraints $B \overset{\mathcal{M}}{\equiv} \emptyset$ for all $B \subset A$. The introduction of two integrity constraints on $A, B \in D^\Theta$ implies the constraint $(A \cup B) \in D^\Theta \equiv \emptyset$ and this implies the emptiness of all $C \in D^\Theta$ such that $C \subset (A \cup B)$. The Shafer's model $\mathcal{M}^0(\Theta)$, can be considered as the most constrained DSm hybrid model including all possible exclusivity constraints *without non-existential constraint*, since all elements in the frame are forced to be mutually exclusive.

2.4 DSm rule of combination for hybrid models

The DSm hybrid rule of combination, associated to a given DSm hybrid model $\mathcal{M} \neq \mathcal{M}_\emptyset$, for $k \geq 2$ independent sources of information is defined for all $A \in D^\Theta$ as [14]:

$$m_{\mathcal{M}(\Theta)}(A) \triangleq \phi(A)\Big[S_1(A) + S_2(A) + S_3(A)\Big] \qquad (2)$$

where $\phi(A)$ is the characteristic emptiness function of a set A, i.e. $\phi(A) = 1$ if $A \notin \boldsymbol{\emptyset}$ and $\phi(A) = 0$ otherwise, where $\boldsymbol{\emptyset} \triangleq \{\boldsymbol{\emptyset}_\mathcal{M}, \emptyset\}$. $\boldsymbol{\emptyset}_\mathcal{M}$ is the set of all elements of D^Θ which have been forced to be empty through the constraints of the model $\mathcal{M}$ and $\emptyset$ is the classical/universal empty set. $S_1(A) \equiv m_{\mathcal{M}^f(\theta)}(A)$, $S_2(A)$, $S_3(A)$ are defined by [14]

$$S_1(A) \triangleq \sum_{\substack{X_1,X_2,\ldots,X_k \in D^\Theta \\ (X_1 \cap X_2 \cap \ldots \cap X_k) = A}} \prod_{i=1}^{k} m_i(X_i) \qquad (3)$$

$$S_2(A) \triangleq \sum_{\substack{X_1,X_2,\ldots,X_k \in \boldsymbol{\emptyset} \\ [\mathcal{U}=A] \vee [(\mathcal{U} \in \boldsymbol{\emptyset}) \wedge (A = I_t)]}} \prod_{i=1}^{k} m_i(X_i) \qquad (4)$$

$$S_3(A) \triangleq \sum_{\substack{X_1,X_2,\ldots,X_k \in D^\Theta \\ (X_1 \cup X_2 \cup \ldots \cup X_k) = A \\ (X_1 \cap X_2 \cap \ldots \cap X_k) \in \boldsymbol{\emptyset}}} \prod_{i=1}^{k} m_i(X_i) \qquad (5)$$

with $\mathcal{U} \triangleq u(X_1) \cup u(X_2) \cup \ldots \cup u(X_k)$ where $u(X)$ is the union of all singletons θ_i that compose X. $S_1(A)$ corresponds to the classic DSm rule of combination based on the free-DSm model; $S_2(A)$ represents the mass of all relatively and absolutely empty sets which is transferred to the total or relative ignorances; $S_3(A)$ transfers the sum of relatively empty sets to the non-empty sets.

3 Basic Elements of Tracking Process

The tracking process consists of two basic elements: *data association* and *track filtering*. The first element is often considered as the most important. Its goal is to associate observations to existing tracks.

3.1 Data Association

To eliminate unlikely observation-to-track pairing at the begining a validation region (gate) is formed around the predicted track position. A measurements in the gate are candidates for association to the corresponding track.

3.1.1 Gating

We assume zero-mean Gaussian white noise for measurements. The vector difference between received measurement vector $\mathbf{z}_j(k)$ and predicted measurement vector $\hat{\mathbf{z}}_i(k|k-1)$ of target i is defined to be residual vector (called innovation) $\tilde{\mathbf{z}}_{ij}(k) = \mathbf{z}_j(k) - \hat{\mathbf{z}}_i(k|k-1)$ with residual covariance matrix $\mathbf{S} = \mathbf{HPH}' + \mathbf{R}$, where $\mathbf{P}$ is the state prediction covariance matrix, $\mathbf{H}$ is the measurement matrix and $\mathbf{R}$ is the measurement covariance matrix [1, 2, 3, 4]. The scan indexes k will be dropped for notational convenience. The norm (normalized distance function) of the innovation is evaluated as $d_{ij}^2 = \tilde{\mathbf{z}}_{ij}'\mathbf{S}^{-1}\tilde{\mathbf{z}}_{ij}$. One defines a threshold constant for gate γ such that correlation is allowed if the following relationship is satisfied

$$d_{ij}^2 \leq \gamma \tag{6}$$

Assume that the measurement vector size is M. The quantity d_{ij}^2 is the sum of the squares of M independent Gaussian random variables with zero means and unit standard deviations. For that reason d_{ij}^2 will have χ_M^2 distribution with M degrees of freedom and allowable probability of a valid observation falling outside the gate. The threshold constant γ can be defined from the table of the chi-square (χ_M^2) distribution [2].

3.1.2 Generalized Data Association

If a single observation is within a gate and if that observation is not within a gate of any other track, the observation can be associated with this track and used to update the track filter. But in a dense target environment additional logic is required when an observation falls within the gates of multiple target tracks or when multiple observations fall within the gate of a target track.

When attribute data are available, the generalized probability can be used to improve the assignment. In view of independence of the kinematic and attribute measurement errors, the generalized probability for measurement j originating from track i is:

$$P_{\text{gen}}(i,j) = P_{\text{k}}(i,j)P_{\text{a}}(i,j)$$

where $P_{\text{k}}(i,j)$ and $P_{\text{a}}(i,j)$ are kinematic and attribute probability terms respectively.

Our goal is to choose a set of assignments $\{\chi_{ij}\}$, for $i = 1, \ldots n$ and $j =, \ldots, m$, that assures maximum of the total generalized probability sum. To find it, we use the solution of the assignment problem

$$\min \sum_{i=1}^{n} \sum_{j=1}^{m} a_{ij}\chi_{ij}$$

where:

$$\chi_{ij} = \begin{cases} 1 & \text{if measurement } j \text{ is assigned to track } i \\ 0 & \text{otherwise} \end{cases}$$

Because our probabilities vary $0 \leq P_{\text{k}}(i,j), P_{\text{a}}(i,j) \leq 1$ and to satisfy the condition to be minimized, the elements of the particular assignment matrix are defined as :

$$a_{ij} = 1 - P_{\text{gen}}(i,j) = 1 - P_{\text{k}}(i,j)P_{\text{a}}(i,j)$$

3.2 Filtering

The used tracking filter is the first order extended Kalman filter [7] for *target state vector* $\mathbf{x} = [x\,\dot{x}\,y\,\dot{y}]'$, where x and y are Cartesian coordinates and $\dot{x}$ and $\dot{y}$ are velocities along Cartesian axes and *measurement vector* $\mathbf{z} = [\beta\,D]'$, where β is the azimuth (measured from the North), and D is the distance from the observer to the target under consideration.

The measurement function $\mathbf{h}(.)$ is (assuming the sensor located at position (0,0)):

$$\mathbf{h}(\mathbf{x}) = [h_1(\mathbf{x})\,h_2(\mathbf{x}]' = [\arctan(\frac{x}{y})\,\sqrt{x^2+y^2}]'$$

and the Jacobian [2]:

$$\mathbf{H} = [H_{ij}] = [\partial h_i/\partial x_j] \quad i = 1,2 \quad j = 1, \ldots, 4$$

We assume constant velocity target model. The process noise covariance matrix is: $\mathbf{Q} = \sigma_v^2\mathbf{Q}_T$, where T is the sampling/scanning period, σ_v is standard deviation of the process noise and $\mathbf{Q}_T$ is given by [8]:

$$\mathbf{Q}_T = diag(\mathbf{Q}_{2\times2}, \mathbf{Q}_{2\times2}) \quad \text{with} \quad \mathbf{Q}_{2\times2} = \begin{bmatrix} \frac{T^4}{4} & \frac{T^3}{2} \\ \frac{T^3}{2} & T^2 \end{bmatrix}$$

The measurement error matrix is $\mathbf{R} = diag(\sigma_\beta^2, \sigma_D^2)$ where σ_β and σ_D are the standard deviations of measurement errors for azimuth and distance.

The track initiation is performed by two-point differencing [7]. After receiving observations for first two scans the initial state vector is estimated by $\hat{\mathbf{x}} = [x(2)\,\frac{x(2)-x(1)}{T}\,y(2)\,\frac{y(2)-y(1)}{T}]'$ where $(x(1), y(1))$ and $(x(2), y(2))$ are respectively the target positions at the first scan for time stamp $k = 1$, and at the second scan for $k = 2$. The initial (starting at time stamp $k = 2$) state covariance matrix $\mathbf{P}$ is evaluated by:

$$\mathbf{P} = diag(\mathbf{P}_{2\times2}^x, \mathbf{P}_{2\times2}^y) \quad \text{with} \quad \mathbf{P}_{2\times2}^{(.)} = \begin{bmatrix} \sigma_{(.)}^2 & \frac{\sigma_{(.)}^2}{T} \\ \frac{\sigma_{(.)}^2}{T} & \frac{2\sigma_{(.)}^2}{T^2} \end{bmatrix}$$

where the index $(.)$ must be replaced by either x or y indexes with $\sigma_x^2 \approx \sigma_D^2 \sin^2(z_\beta) + z_D^2 \sigma_\beta^2 \cos^2(z_\beta)$ and $\sigma_y^2 \approx \sigma_D^2 \cos^2(z_\beta) + z_D^2 \sigma_\beta^2 \sin^2(z_\beta)$. z_β and z_D are the components of the measurement vector received at scan $k = 2$, i.e. $\mathbf{z} = [z_\beta \, z_D]' = \mathbf{h}(\mathbf{x}) + \mathbf{w}$ with $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \mathbf{R})$.

4 The Attribute Contribution to Generalized Data Association

Data association with its goal of partitioning observations into tracks is a key function of any surveillance system. An advanced tendency is the incorporation of generalized data (kinematics and attribute) association to improve track maintenance performance in complicated situations, when kinematics data are insufficient for coherent decision making process. Analogously with the kinematic tracking, the attribute tracking can be considered as the process of combining information collected over time from one or more sensors to refine the knowledge about the evolving attributes of the targets. The motivation for attribute fusion is inspired from the necessity to ascertain the targets types, information, that in consequence has an important implication to enhance the tracking performance. A number of techniques, probabilistic in nature are available for attribute fusion. Their analysis led us to belief, that the theory of Dempster-Shafer is well suited for representing uncertainty, but especially in case of low conflicts between the bodies of evidence. When the conflict increases and becomes high, (case, which often occurs in data association process) the combinational rule of Dempster hides the risk to produce indefiniteness. To avoid that significant risk we considers the form of attribute likelihood function within the context of DSm theory, i.e. the term to be used for computing the probabilities of validity for data association hypotheses. There are a few basic steps, realizing the concept of attribute data association.

4.1 The Input Fuzzification Interface

Fuzzification interface (see fig. 1) transforms numerical measurement received from a sensor into fuzzy set in accordance with the a priori defined fuzzy partition of input space-the frame of discernments Θ. This frame includes all considered linguistic values related to the chosen particular input variable and corresponding membership functions. The fuzzification of numerical sensory data needs dividing an optimal membership into a suitable number of fuzzy sets [17]. Such division provides smooth transitions and overlaps among the associated fuzzy sets, according to the particular real world situation.

The considerable input variable in the particular case is the Radar Cross Section (RCS) of the observed targets. In our work RCS data are analyzed to determine the target size with the subsequent declaration that the observed target is an aircraft of specified type. Taking it in mind, we define two frames of discernments: first one according to the **size of RCS**: $\Theta_1 = \{\text{(S)mall}, \text{(B)ig}\}$ and the second one determining the corresponding to its **Target Type** $\Theta_2 = \{\text{(F)ighter}, \text{(C)argo}\}$.

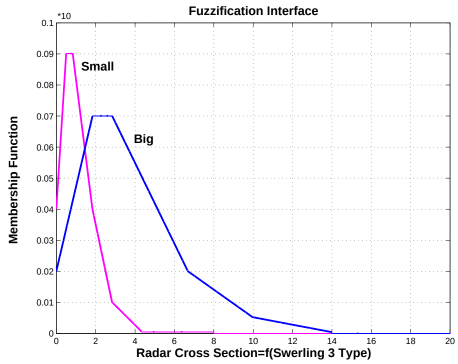

Fig. 1: Fuzzification interface

The radar cross section is modelled as Swerling 3 type, where the density function for the RCS σ is given by:

$$f(\sigma) = \frac{4\sigma}{\sigma_{\text{ave}}^2} \exp[-\frac{2\sigma}{\sigma_{\text{ave}}}]$$

with the average RCS (σ_{ave}) varying between different targets types [6]. The cumulative distribution function of the radar cross section is given by

$$F(\sigma_0) = P\{0 \leq \sigma \leq \sigma_0\} = 1 - (1 + \frac{2\sigma_0}{\sigma_{\text{ave}}}) \exp[-\frac{2\sigma_0}{\sigma_{\text{ave}}}]$$

Since the probabilities $F(\sigma_0)$ for having different values of radar cross section are uniformly distributed in the interval $[0, 1]$ over time (i.e. these values are uncorrelated in time), a sample of observation of the RCS can be simulated by solving equation:

$$(1 + \frac{2\sigma_0}{\sigma_{\text{ave}}}) \exp[-\frac{2\sigma_0}{\sigma_{\text{ave}}}] = 1 - x$$

where x is a random number that is uniformly distributed between 0 and 1.

The scenario considered in our work deals with targets types Fighter (F) and Military Cargo (C) with an average RCS :

$$\sigma_{\text{ave}}^{\text{F}} = 1.2\, m^2 \qquad \text{and} \qquad \sigma_{\text{ave}}^{\text{C}} = 4\, m^2$$

The input fuzzification interface maps the current modelled RCS values into two fuzzy sets: **Small** and **Big**, which define the corresponding linguistic values, defining the variable RCS. Their membership functions are not arbitrarily chosen, but rely on the calculated respective histograms for 10000 Monte Carlo runs. Actually these fuzzy sets form Θ_1 frame of discernement. After fuzzification the new RCS value (rcs) is obtained in the form :

$$\text{rcs} \Rightarrow [\mu_{\text{Small}}(\text{rcs}), \mu_{\text{Big}}(\text{rcs})]$$

In general, the terms $\mu_{\text{Small}}(\text{rcs})$, $\mu_{\text{Big}}(\text{rcs})$ represent the possibilities the new RCS value to belong to the elements of the frame Θ_1 and there is no requirement to sum up to unity.

4.2 Tracks Updating Procedures

4.2.1 Using Classical DSm Combinational Rule

After receiving the new observations, detected during the current scan at time k, DSm classical combinational rule is used for tracks updating. The gbbas of tracks histories and new observations are described in terms of the hyper-power set $D^{\Theta_1} = \{\mathrm{S}, \mathrm{B}, \mathrm{S} \cap \mathrm{B}, \mathrm{S} \cup \mathrm{B}\}$.

To obey the requirements to guarantee that $m(.)$ is a proper general information granule, it is necessarily to transform fuzzy membership functions representing the new measurement into mass functions, before being fused. It is realized through their normalization with respect to the unity:

$$m_{\text{meas}}(C) = \frac{\mu_C(\text{rcs})}{\sum_{C \in \Theta_1} \mu_C(\text{rcs})}, \quad \forall C \in \Theta_1 = \{\mathrm{S}, \mathrm{B}\}$$

Using the classical DSm rule of combination, the updated tracks gbba become:

$$\begin{aligned} m^{ij}_{\text{upd}}(C) &= [m^i_{\text{hist}} \oplus m^j_{\text{meas}}](C) \\ &= \sum_{A,B \in D^{\Theta_1}, A \cap B = C} m^i_{\text{hist}}(A) m^j_{\text{meas}}(B) \end{aligned}$$

where $m^{ij}_{\text{upd}}(.)$ represents the gbba of the updated tracks i with the new observation j; m^i_{hist}, m^j_{meas} are respectively gbba vectors of tracks i history and the new observation j.

Since, DSmT uses a frame of discernment, which is exhaustive, but in general case not exclusive, we are able to take into account and to utilize the paradoxical information $\mathrm{S} \cap \mathrm{B}$. This information relates to the case, when the RCS value resides in an overlapping region, when it is hard to make proper judgement about the tendency of its value. But nevertheless this nonempty set and related to it mass assignment contributes to a better understanding of the overall process.

4.2.2 Using DSm Hybrid Combinational Rule

As it was mentioned above in our work, RCS data here are used to analyze and subsequently to determine the specified type of the observed targets. Because of this it is maintained the second frame of discernement $\Theta_2 = \{\text{(F)ighter}, \text{(C)argo}\}$, in terms of which the decisions according to target types have to be made. The corresponding hyper-power set should be $D^{\Theta_2} = \{\mathrm{F}, \mathrm{C}, \mathrm{F} \cap \mathrm{C}, \mathrm{F} \cup \mathrm{C}\}$. Taking in mind the correspondencies:

- If rcs is **Small** *then* the target is **Fighter**
- If rcs is **Big** *then* the target is **Cargo**

we may transform the gbba of updated tracks, formed in D^{Θ_1} into respective gbba in D^{Θ_2}, i.e:

$$m^{ij}_{\text{upd}}(C_{C \in D^{\Theta_2}}) = m^{ij}_{\text{upd}}(C_{C \in D^{\Theta_1}})$$

This equation contains, among all its elements, the following one:

$$m^{ij}_{\text{upd}}(\mathrm{F} \cap \mathrm{C}) = m^{ij}_{\text{upd}}(\mathrm{S} \cap \mathrm{B})$$

And while the set $\mathrm{S} \cap \mathrm{B} \neq \emptyset$, the set $\mathrm{F} \cap \mathrm{C} = \emptyset$, because it is a proven fact, that the target can not be in one and the same time Fighter and Cargo. So, we have to update the previous fusion result with this new information on the model of the considered problem. It is solved with the DSm hybrid rule, which transfers the mass of that empty set to the non-empty sets of D^{Θ_2}. Using the DSm hybrid rule (2) **with the exclusivity constraint** $\mathrm{F} \cap \mathrm{C} \overset{\mathcal{M}_2}{\equiv} \emptyset$, we get:

$$m^{ij}_{\text{upd}}(\mathrm{F} \cap \mathrm{C}) = 0$$

$$m^{ij}_{\text{upd}}(\mathrm{F}) = m^{ij}_{\text{upd}}(\mathrm{S}) \qquad m^{ij}_{\text{upd}}(\mathrm{C}) = m^{ij}_{\text{upd}}(\mathrm{B})$$

$$m^{ij}_{\text{upd}}(\mathrm{F} \cup \mathrm{C}) = m^{ij}_{\text{upd}}(\mathrm{S} \cap \mathrm{B}) + m^{ij}_{\text{upd}}(\mathrm{S} \cup \mathrm{B})$$

It is important to note, that for us the two considered independent sources of information are the tracks histories and the new observations with their gbbas maintained in terms of the two hyper-power sets. That way, we assure to obtain and to keep the decisions according to the target types during all the scans.

5 The Generalized Data Association Algorithm (GDA)

We assume the existence of a set of n tracks at the current scan and received set of m observations. These observations may be used for updating the existing tracks or for initiating new tracks. In a cluttered environment m does not necessarily equal n and it may be difficult to distinguish whether a measurement originated from a target or from clutter. A validated measurement is one which is either inside or on the boundary of the validation gate of a target. The inequality given in (6) is a validation test. It is used for filling the assignment matrix $\mathbf{A}$:

$$\mathbf{A} = [A_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \vdots & a_{1m} \\ a_{21} & a_{22} & a_{23} & \vdots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \vdots & a_{nm} \end{bmatrix}$$

The elements of the assignment matrix $\mathbf{A}$ have the following values [15]:

$$a_{ij} = \begin{cases} \infty & \text{if } d^2_{ij} > \gamma \\ 1 - P_{\text{k}}(i,j) P_{\text{a}}(i,j) & \text{if } d^2_{ij} \leq \gamma \end{cases}$$

The solution of the assignment matrix is the one that minimizes the sum of the choosen elements. We solve the assignment problem by realizing the extension of Munkres algorithm, given in [9]. As a result, it obtains the optimal measurements to tracks association. Because of the considered closely spaced target scenario, to produce the probability terms P_{k} and P_{a}, the joint probabilistic approach is used [7]. It assures a common base for their defining, making that way them to be compatible.

5.1 Kinematics probability term for GDA

Let consider the existence of two tracks and two new observations, detected during the moment of their closely spaced movement. The table 1 shows the hypotheses for the alternatives with respect to targets tracks and associated probabilities:

Hyp. #	Track 1	Track 2	Hyp. proba. $P'(H_l)$
H_1	0	0	$(1-P_d)^2\beta^2$
H_2	1	0	$g_{11}P_d(1-P_d)\beta$
H_3	2	0	$g_{12}P_d(1-P_d)\beta$
H_4	0	1	$g_{21}P_d(1-P_d)\beta$
H_5	0	2	$g_{22}P_d(1-P_d)\beta$
H_6	1	2	$g_{11}g_{22}P_d^2$
H_7	2	1	$g_{12}g_{21}P_d^2$

Table 1: Target-oriented hypothesis based on kinematics.

Here, the numbers in the second and third columns of the table represent the observations assigned to the tracks, and 0 represents the assignment of no observation to a particular track. The likelihood function, associated with the assignment of observation j to track i is:

$$g_{ij} = \frac{e^{-d_{ij}^2/2}}{(2\pi)^{M/2}\sqrt{|\mathbf{S}_i|}}$$

P_d is the probability of detection and β is the extraneous return density, that includes probability density for new tracks and false alarms: $\beta = \beta_{NT} + \beta_{FA}$. The hypotheses probabilities are computed as :

$$P'(H_l) = \beta^{N_M-(N_T-N_{nD})}(1-P_d)^{N_{nD}} \times {P_d}^{(N_T-N_{nD})} \prod_{i\neq 0, j\neq 0|(i,j)\in H_l} g_{ij}$$

N_M being the total number of observations, N_T the number of targets, N_{nD} the number of not detected targets. $(i,j)\in H_l$ involved in the product represents all the possible measurement to track associations involved within hypothesis H_l. The normalized probabilities of association are computed as:

$$P_{\text{k}}(H_l) = \frac{P'(H_l)}{\sum_{k=1}^{N_H} P'(H_k)}$$

where N_H is the number of possible association hypotheses. To compute the probability $P_{\text{k}}(i,j)$ that observation j should be assigned to track i, a sum is taken over the probabilities $P_{\text{k}}(.)$ from those hypotheses H_l, in which this assignment occurs.

5.2 Attribute probability terms for GDA

The way of calculating the attribute probability term follows the joint probabilistic approach. In the case of existence of two tracks and two new observations, considered in 5.1 and on the base of the hypotheses matrix (Table1) one can obtain the respective Euclidean distances $d_e(ij)$ between gbbas of each pair (tracks history i - observation j) as a measures of their closeness.

Hyp. #	Track 1	Track 2	Closeness measure
H_1	0	0	$P''(H_1) = d_e(0,0) = 0$
H_2	1	0	$P''(H_2) = d_e(1,1)$
H_3	2	0	$P''(H_3) = d_e(1,2)$
H_4	0	1	$P''(H_4) = d_e(2,1)$
H_5	0	2	$P''(H_5) = d_e(2,2)$
H_6	1	2	$P''(H_6) = d_e(1,1)d_e(2,2)$
H_7	2	1	$P''(H_7) = d_e(1,2)d_e(2,1)$

Table 2: Target-oriented hypothesis based on attributes.

where the overal measure of closeness $d_e(ij)$ is defined here for $i\neq 0$ and $j\neq 0$ as

$$d_e(ij) = \sqrt{\sum_{C\in D^{\Theta_2}} \left[m^i_{\text{hist}}(C) - m^j_{\text{meas}}(C)\right]^2}$$

m^i_{hist} is the gbba of the tracks history, m^j_{meas} is the gbba of measurement j. The corresponding normalized probabilities of association drawn from attribute information are then obtained as:

$$P_{\text{a}}(H_l) = \frac{P''(H_l)}{\sum_{k=1}^{N_H} P''(H_k)}$$

where N_H is the number of possible association hypotheses. To compute the probability $P'_{\text{a}}(i,j)$ that observation j should be assigned to track i, a sum is taken over the probabilities $P_{\text{a}}(.)$ from those hypotheses H_l, in which this assignment occurs. Because the Euclidean distance is inversely proportional to the probability of association, the probability term $P_{\text{a}}(i,j) = 1 - P'_{\text{a}}(i,j)$ is used to match the corresponding kinematics probability.

6 Simulation scenario

The simulation scenario consists of two air targets (Fighter and Cargo) and a stationary sensor at the origin with $T_{\text{scan}} = 5$ sec., measurement standard deviations 0.3 deg and 60 m for azimuth and range respectively. The targets movement is from West to East with constant velocity of 250 m/sec. The headings of the fighter and cargo are 225 deg and 315 deg from North respectively. During the scan 11th-14th the targets perform maneuvers with 2.5g. Their trajectories are closely spaced in the vicinity of the two crossing points. The target detection probabilities have been set to 0.99 for both targets and the extraneous return density β to 10^{-6}. In this scenario we do not consider the more complicated situations, when the false alarms are available. This case has been also analyzed and is presented in [14].

7 Simulation results

In this section the obtained simulation results, based on 500 Monte Carlo runs are presented. The goal is to demonstrate how the attribute measurements contributes for improvement the track performance, especially in critical cases, when the two tracks are closely spaced.

In the KDA case on fig. 2 (i.e. Kinematics-based Data Association), it is evident that after scan 15 (the second crossing moment for the targets), the tracking algorithm loses the proper targets direction. Here the *Tracks Purity* performance criterion is used to examine the ratio of the right associations. Track purity is considered as a ratio of the number of correct observation-target associations (in case of detected target) over the total number of available observations during tracking scenario. As it is obvious from table 3, 0.80 of (observation-track) associations are the proper ones in that case. Figure 3 shows the result, when

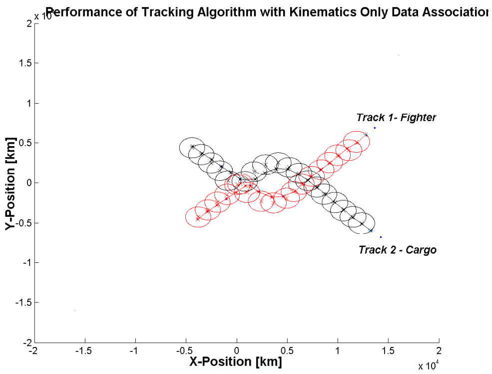

Fig. 2: Performance of Tracking with KDA

attribute data are utilized in the generalized data association algorithm in order to improve the tracks maintenance performance. The DSm hybrid rule is applied to produce the attribute probability term in generalized assignment matrix. As a result it is obvious from table 4 that the tracks purity increases up to 0.9367.

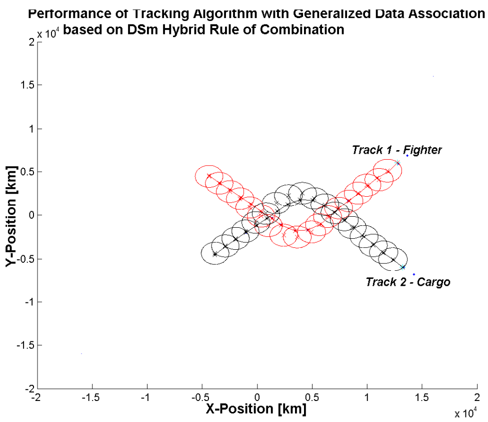

Fig. 3: Performance of Tracking with GDA

	Obs. 1	Obs. 2
Tr. 1	0.8	0.2
Tr. 2	0.2	0.8

Table 3: Tracks' purity in case of KDA.

	Obs. 1	Obs. 2
Tr. 1	0.9367	0.0633
Tr. 2	0.0633	0.9367

Table 4: Tracks' purity in case of GDA.

Figure 4 represents the probability of mis-correlation. After scan 12, just when begin the targets maneouvers, it is evident that in case of kinematics only data association the mis-correlation probability increases up to 0.6. The attribute utilization leads to decreasing of mis-correlation probability to 0.2. The following two figures show the

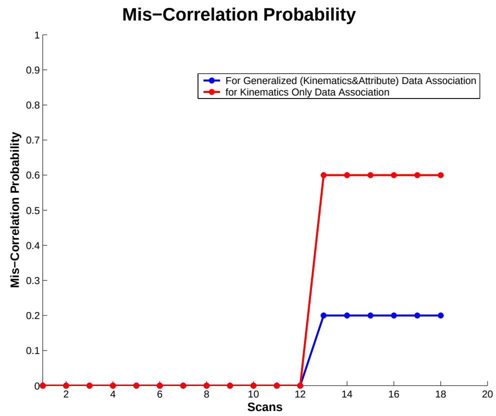

Fig. 4: Performance of Tracking Algorithm with GDA

variations of measured Pignistic entropy in updated tracks attribute states, presented in the two hyper-power sets : $D^{\Theta_1} = \{\mathrm{S}, \mathrm{B}, \mathrm{S}{\cap}\mathrm{B}, \mathrm{S}{\cup}\mathrm{B}\}$ and $D^{\Theta_2} = \{\mathrm{F}, \mathrm{C}, \mathrm{F}{\cap}\mathrm{C}, \mathrm{F}{\cup}\mathrm{C}\}$. It can be seen that :

- The pignistic entropy of the right (track-observation) associations is less than the pignistic entropy of the wrong ones (see fig. 5 and 6);
- In the cases of right associations, it is obvious that in general the pignistic entropy is decreasing in the cases when the tracks updating is realized in $D^{\Theta_1} = \{\mathrm{S}, \mathrm{B}, \mathrm{S}\cap\mathrm{B}, \mathrm{S}\cup\mathrm{B}\}$ than in the hyper powerset $D^{\Theta_2} = \{\mathrm{F}, \mathrm{C}, \mathrm{F}\cap\mathrm{C}, \mathrm{F}\cup\mathrm{C}\}$. It is because of the integrity constraint $\mathrm{F}\cap\mathrm{C} \overset{\mathcal{M}_2}{\equiv} \emptyset$ (presented in section 4.2.2), which cause the mass transfer to the uncertainty according to $m^{ij}_{\text{upd}}(\mathrm{F}\cap\mathrm{C}) = 0$, $m^{ij}_{\text{upd}}(\mathrm{F}) = m^{ij}_{\text{upd}}(\mathrm{S})$, $m^{ij}_{\text{upd}}(\mathrm{C}) = m^{ij}_{\text{upd}}(\mathrm{B})$ and $m^{ij}_{\text{upd}}(\mathrm{F}{\cup}\mathrm{C}) = m^{ij}_{\text{upd}}(\mathrm{S}{\cap}\mathrm{B}) + m^{ij}_{\text{upd}}(\mathrm{S}{\cup}\mathrm{B})$.

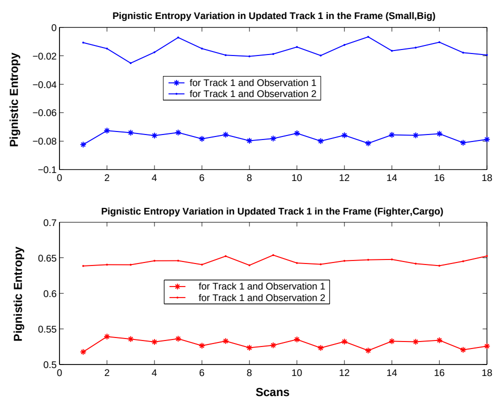

Fig. 5: Variation of Pignistic Entropy in Track 1 Attribute State in the $\Theta_1 = \{(\mathrm{S})\text{mall}, (\mathrm{B})\text{ig}\}$ and $\Theta_2 = \{(\mathrm{F})\text{ighter}, (\mathrm{C})\text{argo}\}$

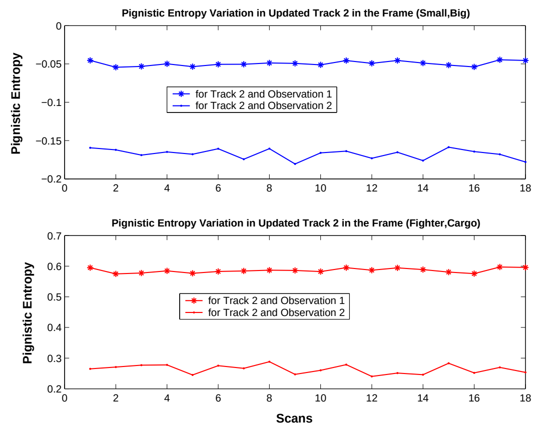

Fig. 6: Variation of Pignistic Entropy in Track 2 Attribute State in the $\Theta_1 = \{(\mathrm{S})\mathrm{mall}, (\mathrm{B})\mathrm{ig}\}$ and $\Theta_2 = \{(\mathrm{F})\mathrm{ighter}, (\mathrm{C})\mathrm{argo}\}$

8 Conclusions

In this paper a new approach for target tracking, which incorporates the advanced concept of generalized data (kinematics and attribute) association is presented. The realized algorithm is based on Global Nearest Neighbour-like approach and uses Munkres algorithm to resolve the generalized association matrix. The principles of Dezert-Smarandache theory (DSmT) of plausible and paradoxical reasoning to utilize attribute data are applied. Especially the new general DSm hybrid rule of combination is used to deal with particular integrity constraints associated with some elements of the free Dedekind's distributive lattice. As a result of applied tracking algorithm the improvement of track maintenance performance in complicated situations (closely spaced targets) is realized, assuring a coherent decision making, when kinematics data are insufficient to provide the proper decisions. A comparison of this new GDA approach with the recent Feature/Class-Augmented Likelihood-Function-based Data Association method (F/CA-LF DA) [5] is under investigation.

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