Multisensor-Multitarget Bias Estimation for General Asynchronous Sensors

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Abstract – *This paper provides the exact solution for the bias estimation problem in multiple asynchronous sensors using common targets of opportunity. The target data reported by the sensors are usually not time-coincident or synchronous due to the different data rates. Since the bias estimation requires time-coincident target data from different sensors, a novel scheme is used to transform the measurements from the different times of the sensors into pseudomeasurements of the sensor biases with additive noises that are zero-mean, white and with easily calculated covariances. These results allow bias estimation as well as the evaluation of the Cramer-Rao Lower Bound (CRLB) on the covariance of the bias estimate, i.e., the quantification of the available information about the biases in any scenario. Monte Carlo simulation results show that the new method is statistically efficient, i.e., it meets the CRLB.*

Keywords: Sensor bias estimation, registration, multisensor-multitarget tracking, asynchronous sensors, multisensor fusion.

1 Introduction

Registration error correction is vital in multiple sensor systems in order to carry out data fusion. This requires estimation of the sensor measurement biases. It is important to correct for these bias errors so that the multiple sensor measurements and/or tracks can be referenced to a common tracking coordinate system (frame). If uncorrected, registration error can lead to large tracking errors and potentially to the formation of multiple tracks (ghosts) on the same target.

In this paper, we consider a multiple sensor tracking system with a decentralized information processing architecture. Each local tracker obtains its own local state estimates using local measurements and transmits these estimates to the fusion center. The fusion center performs track-to-track fusion on demand after estimating these sensor biases based on the common targets being tracked by the different sensors (see, e.g, [5]). The transmission can be also on demand, i.e., no "full-rate" communication is necessary.

To estimate the bias vector, the classical approach is to augment the system state to include the bias vector as part of the state, and then implement an augmented state Kalman filter (ASKF) by stacking the state of all the targets and the sensor biases into a single vector. The problem with this approach is that the implementation of this ASKF can be computationally infeasible. In addition, numerical problems may arise during the implementation; mainly, for ill-conditioned systems. Friedland [12] proposed the idea of implementing two parallel, reduced-order filters instead of the use of a ASKF. Alouani, Rice and Blair [1] showed that under a restrictive algebraic constraint, the optimal estimate of the state can be obtained as a linear combination of the outputs of the local bias-ignorant estimate and the bias estimate. They claimed that since the algebraic constraint can be too restrictive in practice, that is an indirect proof why all practical two-stage filters are suboptimal. Van Doorn and Blom [29] gave the exact solution for the augmented Kalman filter problem but then decoupled the equations using an approximation in order to make the implementation feasible. A similar approach was used in [18, 28] to separate the tracker from the bias filter and it undoubtedly imposed some (uncharacterized) loss in estimation performance. Okello and Ristic's recent work in [26] is a batch algorithm that estimates biases by iterating on the inverse of the measurement equation linearized around the latest central estimate as if it was perfect, which leads to a calculated CRLB that is too small. The recursive performance bound in [13] is realistic, but it is obtained using ASKF, which is not computationally efficient.

Lin, Kirubarajan and Bar-Shalom [19] presented the bias estimation based on the local unassociated track estimates at a single time, i.e., based on a single frame. In [20], the authors extended the work of [19] to include the dynamic bias estimation based on the local track estimates at different times, i.e., based on multiple frames. The technique in [20] has been shown to yield absolute registration for the biases subject to an observability condition. The work of [21] extended [19] and [20] to a bias model which considers both offset biases and scale biases and showed some preliminary results.

However, in reality, the target data reported by the sensors are usually not time-coincident or synchronous due to the different data rates. The bias estimation of asynchronous sensors does not seem to be understood or even widely recognized. All of these above papers as well as other well-known papers on sensor bias estimation, such as [8, 9, 10, 25], are considering only synchronous sensors. In [15], authors extended the work of [14] to asynchronous sensors by using one-step fixed-lag IMM predictor to translate the estimates to common time, and only relative registration was achieved under a relatively restrictive assumption that the distance between two sensors is small. The situation of asynchronous sensors can be handled by the ASKF, but for a large number of targets it becomes infeasible.

In [22], the bias estimation for asynchronous sensors with same sampling rate but with a phase offset is discussed. The purpose of this paper is to extend the work of [19], [20], [21] and [22] to general asynchronous sensors, which makes it applicable for practical systems. A novel scheme is proposed to transform the measurements from the different times into the pseudomeasurements such that the pseudomeasurement noises of the sensor biases are zero-mean, white and with easily calculated covariances. Another significant benefit of this scheme is that the bias estimation is decoupled from the state estimation without any approximation.

This paper is organized as follows. The bias model and the assumptions for bias estimation are discussed in Section 2. The bias estimation of asynchronous sensors is proposed in Section 3. Section 4 presents the performances of the bias estimators in synchronous sensors and asynchronous sensors for some typical scenarios. It is shown that the proposed technique achieves, similarly to [19], [20], [21] and [22], absolute registration. Conclusions are in Section 5.

2 Bias Model

Consider M sensors which measure the range and azimuth for N common targets in the surveillance region. The model for the biased measurements in polar coordinates for sensor i at time t_j is

$$z_i^p(t_j) = \begin{bmatrix} r_i(t_j) \\ \theta_i(t_j) \end{bmatrix} \tag{1}$$

where the range and azimuth at time k are

$$\begin{aligned} r_i(t_j) &= [1+\epsilon_i^r(t_j)]\, r_i^t(t_j) + b_i^r(t_j) + w_i^r(t_j) \quad (2) \\ \theta_i(t_j) &= \left[1+\epsilon_i^\theta(t_j)\right] \theta_i^t(t_j) + b_i^\theta(t_j) + w_i^\theta(t_j) \quad (3) \end{aligned}$$

In the above $r_i^t(t_j)$ and $\theta_i^t(t_j)$ are the true range and azimuth; $b_i^r(t_j)$ and $b_i^\theta(t_j)$ are the offset biases for the range and azimuth; $\epsilon_i^r(t_j)$ and $\epsilon_i^\theta(t_j)$ are the scale biases of the range and azimuth, respectively; the measurement noises $w_i^r(t_j)$ and $w_i^\theta(t_j)$ are zero-mean, white with corresponding variances σ_r^2 and σ_θ^2 and are assumed mutually independent of each other.

Denote the bias vector for sensor i at time t_j as

$$\beta_i(t_j) \triangleq \begin{bmatrix} b_i^r(t_j) \\ b_i^\theta(t_j) \\ \epsilon_i^r(t_j) \\ \epsilon_i^\theta(t_j) \end{bmatrix} \tag{4}$$

Then,

$$z_i^p(t_j) = \begin{bmatrix} r_i^t(t_j) \\ \theta_i^t(t_j) \end{bmatrix} + C_i(t_j)\beta_i(t_j) + \begin{bmatrix} w_i^r(t_j) \\ w_i^\theta(t_j) \end{bmatrix} \tag{5}$$

where

$$C_i(t_j) \triangleq \begin{bmatrix} 1 & 0 & r_i^t(t_j) & 0 \\ 0 & 1 & 0 & \theta_i^t(t_j) \end{bmatrix} \tag{6}$$

is assumed known — the observed (or estimated) azimuth $\hat{\theta}_i(t_j)$ and range $\hat{r}_i(t_j)$ can be used without any loss of performance.

The problem is to estimate the bias vectors for all sensors. After the bias estimation is complete, the bias estimate can be used to correct the state estimates of the targets. After this, the track-to-track fusion can be performed.

After transforming the measurements into Cartesian coordinates, the measurement equation for sensor i is

$$z_i(t_j) = H(t_j)\mathbf{x}(t_j) + B_i(t_j)C_i(t_j)\beta_i(t_j) + w_i(t_j) \tag{7}$$

where the state vector $\mathbf{x}(t_j) \triangleq [x(t_j) \quad \dot{x}(t_j) \quad y(t_j) \quad \dot{y}(t_j)]'$, $H(t_j)$ is the measurement matrix and the matrix $B_i(t_j)$ is a nonlinear function of the true range and azimuth. If $B_i(t_j)C_i(t_j)$ is constant, only the difference of the biases at different sensors is observable, which means "relative registration" or "incomplete observability". Using the observed (or estimated) azimuth $\hat{\theta}_i(t_j)$ and range $\hat{r}_i(t_j)$ from sensor i, one has the matrix $B_i(t_j)$ in (7) as

$$B_i(t_j) = \begin{bmatrix} \cos\hat{\theta}_i(t_j) & -\hat{r}_i(t_j)\sin\hat{\theta}_i(t_j) \\ \sin\hat{\theta}_i(t_j) & \hat{r}_i(t_j)\cos\hat{\theta}_i(t_j) \end{bmatrix} \tag{8}$$

In (7) $w_i(t_j)$ is the measurement noise with the covariance in the Cartesian coordinates (omitting the time index in the measurements for simplicity)

$$R_i = \begin{bmatrix} r_i^2\sigma_\theta^2 \sin^2\theta_i + \sigma_r^2\cos^2\theta_i & (\sigma_r^2 - r_i^2\sigma_\theta^2)\sin\theta_i\cos\theta_i \\ (\sigma_r^2 - r_i^2\sigma_\theta^2)\sin\theta_i\cos\theta_i & r_i^2\sigma_\theta^2\cos^2\theta_i + \sigma_r^2\sin^2\theta_i \end{bmatrix}$$

Remarks

The polar to Cartesian conversion needed for (7) has been discussed extensively in the literature [5] together with the limit of validity of the standard transformation. If this limit is exceeded, then one can use the modified version [23] which eliminates the bias caused by the nonlinearities in the transformation and also provides the correct covariance. Thus the linear measurement model (7) is exact (in the sense that the noises in it are zero-mean, white and their actual covariance is available), even though it is obtained from a nonlinear coordinate transformation.

The unbiased coordinate conversion of [23] takes care of all practical situations. In view of this, we call our sensor bias estimation technique "exact" because all the assumptions of the technique, namely, the noises in the linear pseudomeasurements of the sensor biases (to be derived in the next section) being zero-mean, white with known covariances, are satisfied exactly.

3 Dynamic Bias Estimation for General Asynchronous Sensors

The target data reported by the sensors are usually not time-coincident or synchronous due to the different data rates and the synchronicity-based algorithm cannot be used for the bias estimation problem of asynchronous sensors. The bias estimation of asynchronous sensors does not seem to be understood or even widely recognized. Most of the well-known papers on sensor bias estimation, such as [8, 9, 10, 25, 26], are considering only synchronous sensors.

The EX method [20] discussed in the previous section requires time-coincident target data from different sensors. Therefore, it is desirable to provide the exact solution for the bias estimation problem in multiple asynchronous sensors using common targets of opportunity. We will extend the EX method of synchronous sensor bias estimation to asynchronous sensors, which will be more realistic and extremely important for practical applications. The extension of the EX method is denoted as the EXX method in the sequel.

Assume the dynamic equation of targets is

$$\mathbf{x}(t_j) = F(t_j, t_i)\mathbf{x}(t_i) + v(t_j, t_i) \tag{9}$$

where we assume[1] the state vector at time t_j is $\mathbf{x}(t_j) \triangleq [x(t_j) \quad \dot{x}(t_j) \quad y(t_j) \quad \dot{y}(t_j)]'$, $F(t_j, t_i)$ is the transition matrix from time t_i to time t_j, and the process noise $v(t_j, t_i) \triangleq [v_x(t_j, t_i) \quad v_{\dot{x}}(t_j, t_i) \quad v_y(t_j, t_i) \quad v_{\dot{y}}(t_j, t_i)]'$ is zero-mean white with covariance $Q(t_j, t_i)$. In this section, we assume the locations of sensors are known.

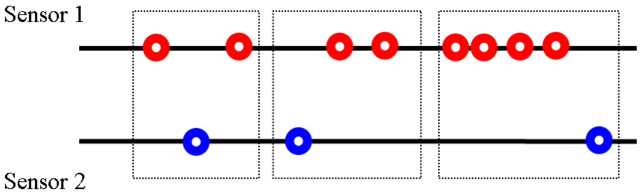

Fig. 1: Asynchronous Measurement Sets and Proper Time Slots

Consider M=2 asynchronous sensors, as shown in Figure 1. The circles represent the measurement sets which are defined as the measurements of all targets of opportunity at a specific time. The data rate of a specific sensor is not necessarily constant, as shown in Figure 1. A "proper time slot" is defined as the minimum interval starting at the time of a particular measurement such that there is a linear combination of all the measurements (from the same target) within the time slot, which is *independent of the target state*. The proper time slots are shown in Figure 1 with dashed boxes.

A proper time slot has the following properties:

- The number of the measurement sets in a proper time slot should be as small as possible. This is because with the smaller number of the measurement sets, the bias estimation can be performed earlier and the updated bias estimates are available earlier. Therefore, a nearly real-time bias estimation can be achieved.

- There should be at least one measurement set from each sensor and at least two measurement sets from one sensor.

A state-independent pseudomeasurement of the biases requires three measurement sets. For n measurement sets in a proper time slot, $n-2$ state-independent pseudomeasurements of the biases will be constructed.

In [22], we considered the two asynchronous sensors with the same sampling rate but with a phase difference. There are exactly three asynchronous measurements in a slot and the slots were used to yield one asynchronous pseudomeasurement equation of the biases. In the case where n is greater than three, there will be $n-2$ asynchronous pseudomeasurement equations of the biases based on these n asynchronous measurements.

Consider one target at this time. When there are more than three measurements in a proper time slot, according to the properties of a proper time slot, the first $n-1$ measurements must be from one sensor, assumed without loss of generality from sensor 1, and the last measurement must be from sensor 2. We denote these asynchronous measurements[2] as $z_b^1(t_1)$, $z_b^1(t_2)$, ..., $z_b^1(t_{n-1})$ and $z_b^2(t_n)$ with $t_1 < t_2 < \ldots < t_{n-1} < t_n$.

For $2 \le j \le n-1$, we can generalize the measurements from sensor 1 based on the true state at time index t_2. Assume the measurement time-invariant matrix $H(t_j)$ and the time-varying transition matrix $F(t_j, t_i)$ are defined as

$$H(t_j) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \triangleq H \tag{10}$$

$$F(t_j, t_i) = \begin{bmatrix} 1 & t_j - t_i & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & t_j - t_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

If we denote the constant matrix as

$$A(j,k) \triangleq \begin{bmatrix} 1 & t_j - t_k & 0 & 0 \\ 0 & 0 & 1 & t_j - t_k \end{bmatrix} \tag{12}$$

we have

$$z_b^1(t_j) = \begin{bmatrix} x(t_{j-2}) + (t_j - t_{j-2})\dot{x}(t_{j-2}) \\ y(t_{j-2}) + (t_j - t_{j-2})\dot{y}(t_{j-2}) \end{bmatrix} + \sum_{k=j-1}^{j} A(j,k) v(t_k, t_{k-1}) +$$

[1] While the discussion is done in this context, the method can be generalized at the expense of much more complicated notation.

[2] We use a single indexing within a proper time slot to avoid cumbersome notations.

$$
\begin{aligned}
&B_1(t_j)C_1(t_j)\beta_1(t_j)+w_1(t_j)\\
= &\begin{bmatrix} x(t_2)+(t_j-t_2)\dot{x}(t_2) \\ y(t_2)+(t_j-t_2)\dot{y}(t_2) \end{bmatrix}+\\
&\sum_{k=3}^{j} A(j,k)v(t_{k-1},t_k)+\\
&B_1(t_j)C_1(t_j)\beta_1(t_j)+w_1(t_j) \qquad (13)
\end{aligned}
$$

The first measurement $z_b^1(t_1)$ can not be included in the above general expression and it has the following expression

$$
\begin{aligned}
z_b^1(t_1) = &\begin{bmatrix} x(t_2)-(t_2-t_1)\dot{x}(t_2) \\ y(t_2)-(t_2-t_1)\dot{y}(t_2) \end{bmatrix}+\\
&A_0 v(t_2,t_1)+B_1(t_1)C_1(t_1)\beta_1(t_1)+w_1(t_1)
\end{aligned} \qquad (14)
$$

where A_0 is a constant matrix with

$$
A_0=\begin{bmatrix} -1 & t_2-t_1 & 0 & 0 \\ 0 & 0 & -1 & t_2-t_1 \end{bmatrix} \qquad (15)
$$

Similarly, we have

$$
\begin{aligned}
z_b^2(t_n) = &\begin{bmatrix} x(t_2)+(t_n-t_2)\dot{x}(t_2) \\ y(t_2)+(t_n-t_2)\dot{y}(t_2) \end{bmatrix}+\\
&\sum_{k=3}^{n} A(n,k)v(t_{k-1},t_k)+\\
&B_2(t_2)C_2(t_2)\beta_2(t_2)+w_2(t_2) \qquad (16)
\end{aligned}
$$

Therefore, $z_b^1(t_1)$, $z_b^1(t_j)$ and $z_b^2(t_n)$ can be used to yield a *state-independent* pseudomeasurement of the biases. Define the pseudomeasurement $z_b(j)$ as the difference of the measurement from sensor 2 and a linear combination of the first and the j-th measurements from sensor 1, that is

$$
z_b(j) \triangleq z_b^2(t_n)-\left[\alpha_1(j)z_b^1(t_1)+\alpha_2(j)z_b^1(t_j)\right] \qquad (17)
$$

then, by appropriately selecting the parameters $\alpha_1(j)$ and $\alpha_2(j)$, the true state of the targets (the position and velocity elements at time t_2) canceled yielding a *state-independent* bias pseudomeasurement. Therefore, the asynchronous bias estimates are decoupled with the state estimates of the targets.

Solving the following two equations

$$
\begin{aligned}
\alpha_1(j)+\alpha_2(j) &= 1 \qquad (18)\\
-(t_2-t_1)\alpha_1(j)+(t_j-t_2)\alpha_2(j) &= t_n-t_2 \qquad (19)
\end{aligned}
$$

we have

$$
\begin{aligned}
\alpha_1(j) &= -\frac{t_n-t_j}{t_j-t_1} \qquad (20)\\
\alpha_2(j) &= \frac{t_n-t_1}{t_j-t_1} \qquad (21)
\end{aligned}
$$

Then

$$
\begin{aligned}
z_b(j) = &B_2(t_n)C_2(t_n)\beta_2(t_n)-\\
&\alpha_1(j)B_1(t_1)C_1(t_1)\beta_1(t_1)-\\
&\alpha_2(j)B_1(t_j)C_1(t_j)\beta_1(t_j)+w_2(t_n)-\\
&\alpha_1(j)w_1(t_1)-\alpha_2(j)w_1(t_j)-\\
&\alpha_1(j)A_0v(t_2,t_1)+\sum_{k=3}^{j} A_1(j,k)v(t_{k-1},t_k)+\\
&\sum_{k=j+1}^{n} A(n,k)v(t_{k-1},t_k) \qquad (22)
\end{aligned}
$$

where $A_1(j,k)$ is a constant matrix with

$$
A_1(j,k) \triangleq A(n,k)-\alpha_2(j)A(j,k) \qquad (23)
$$

If the biases are time-invariant constants, a more compact expression for $z_b(j)$ is

$$
z_b(j)=\mathcal{H}(j)\mathbf{b}+\tilde{w}_b(j) \qquad (24)
$$

where

$$
\begin{aligned}
\mathcal{H}(j) = &\Big[-\alpha_1(j)B_1(t_1)C_1(t_1)-\alpha_2(j)B_1(t_j)C_1(t_j),\\
&B_2(t_n)C_2(t_n)\Big] \qquad (25)\\
\tilde{w}_b(j) = &w_2(t_n)-\alpha_1(j)w_1(t_1)-\alpha_2(j)w_1(t_j)-\\
&\alpha_1(j)A_0v(t_2,t_1)+\sum_{k=3}^{j} A_1(j,k)v(t_{k-1},t_k)+\\
&\sum_{k=j+1}^{n} A(n,k)v(t_{k-1},t_k) \qquad (26)
\end{aligned}
$$

and the bias vector $\mathbf{b}$ is defined as

$$
\mathbf{b} \triangleq \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} \qquad (27)
$$

Note that the pseudomeasurement noises $\tilde{w}_b(j)$ are correlated to each other in a proper time slot.

The stacked form of these $n-2$ asynchronous pseudomeasurements of the biases is

$$
\mathbf{z}_b=\mathcal{H}\mathbf{b}+\tilde{\mathbf{w}}_b \qquad (28)
$$

where the stacked pseudomeasurement vector $\mathbf{z}_b$, the stacked pseudomeasurement matrix $\mathcal{H}$, and the stacked pseudomeasurement noise vector $\tilde{\mathbf{w}}_b$ are defined as

$$
\mathbf{z}_b \triangleq \begin{bmatrix} z_b(2) \\ z_b(3) \\ \cdots \\ z_b(n-1) \end{bmatrix} \qquad (29)
$$

$$
\mathcal{H} \triangleq \begin{bmatrix} \mathcal{H}(2) \\ \mathcal{H}(3) \\ \cdots \\ \mathcal{H}(n-1) \end{bmatrix} \qquad (30)
$$

$$
\tilde{\mathbf{w}}_b \triangleq \begin{bmatrix} \tilde{w}_b(2) \\ \tilde{w}_b(3) \\ \cdots \\ \tilde{w}_b(n-1) \end{bmatrix} \qquad (31)
$$

The asynchronous pseudomeasurement noise of the biases $\tilde{\mathbf{w}}_b$ is zero-mean and white across proper time slots with the covariance $\mathcal{R}_b$. If we denote the diagonal block of the $\mathcal{R}_b$ as $\mathcal{R}_b(j,j)$, and denote the off-diagonal block[3] of $\mathcal{R}_b$ as $\mathcal{R}_b(i,j)$ when $i<j$, we have

$$\begin{aligned}
\mathcal{R}_b(j,j) &\triangleq \text{var}\left[\tilde{w}_b(j)\right] \\
&= R_2(t_n)+\alpha_1(j)^2R_1(t_1)+ \\
&\quad \alpha_2(j)^2R_1(t_j)+\alpha_1(j)^2A_0Q(t_2,t_1)A_0'+ \\
&\quad \sum_{k=3}^{j}A_1(j,k)Q(t_{k-1},t_k)A_1(j,k)'+ \\
&\quad \sum_{k=j+1}^{n}A(n,k)Q(t_{k-1},t_k)A(n,k)' \qquad (32)
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_b(i,j) &\triangleq \text{cov}\left[\tilde{w}_b(i),\tilde{w}_b(j)\right] \\
&= R_2(t_n)+\alpha_1(i)R_1(t_1)\alpha_1(j)+ \\
&\quad \alpha_1(i)A_0Q(t_2,t_1)A_0'\alpha_1(j)+ \\
&\quad \sum_{k=3}^{i}A_1(i,k)Q(t_{k-1},t_k)A_1(j,k)'+ \\
&\quad \sum_{k=i+1}^{j}A(n,k)Q(t_{k-1},t_k)A_1(j,k)'+ \\
&\quad \sum_{k=j+1}^{n}A(n,k)Q(t_{k-1},t_k)A(n,k)' \qquad (33)
\end{aligned}$$

Since across the proper time slots the noises $\tilde{\mathbf{w}}_b$ are white, one can use recursive least squares (RLS) across the proper time slots. Again, like in the case in the previous subsection, the whiteness property of the pseudomeasurement noise is preserved in the stacked asynchronous pseudomeasurement equation (28). Since the state of the target did not appear in the pseudomeasurement equation of the biases, the bias estimation is decoupled from the state estimation. Note that no approximations whatsoever were made in the derivation, i.e., this method is again exact.

4 Simulation Results

Consider a scenario with two asynchronous sensors with different sampling rates. Sensor 1 reports measurements at 1 s interval. Sensor 2 reports measurement at 3 s interval, and there is 2.5 s time offset between the initial reporting times of the two sensors, as illustrated in Figure 2.

The biases for the two sensors are offset and scale biases for range and azimuth,

$$\beta_1=\beta_2=[20\,\text{m} \quad 2\,\text{mrad} \quad 3\times10^{-5} \quad 2\times10^{-4}]' \qquad (34)$$

The geometry of the targets and sensors is shown in Figure 3 and the targets are moving at nearly constant velocity with $\dot{x}=\dot{y}=20\,\text{m/s}$. The standard deviation of the measurement noise variances are $\sigma_r=$ 10 m and $\sigma_\theta=1$ mrad for the range and the azimuth measurements, respectively. Assume the locations of the sensors are known.

[3]The matrix $\mathcal{R}_b$ is symmetric.

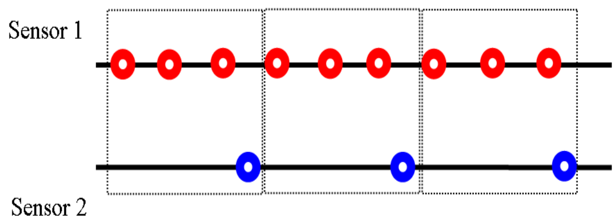

Fig. 2: Asynchronous Sensors with Different Rates

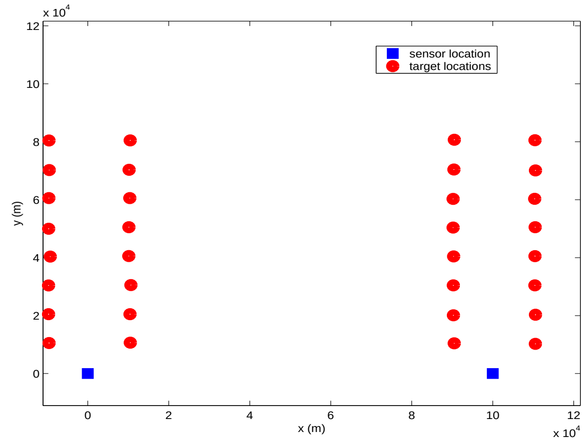

Fig. 3: The Geometry of 32 Targets and Two Asynchronous Sensors

The dynamics of the target are modeled using Discretized Continuous White Noise Acceleration (DCWNA) models [7] where the process noise covariance between time t_i and t_j is

$$Q(t_j,t_i)=\begin{bmatrix} Q_x(t_j,t_i) & 0 \\ 0 & Q_y(t_j,t_i)\end{bmatrix} \qquad (35)$$

with

$$Q_x(t_j,t_i) = \begin{bmatrix} \frac{1}{3}(t_j-t_i)^3 & \frac{1}{2}(t_j-t_i)^2 \\ \frac{1}{2}(t_j-t_i)^2 & (t_j-t_i)\end{bmatrix}\tilde{q}_x \qquad (36)$$

$$Q_y(t_j,t_i) = \begin{bmatrix} \frac{1}{3}(t_j-t_i)^3 & \frac{1}{2}(t_j-t_i)^2 \\ \frac{1}{2}(t_j-t_i)^2 & (t_j-t_i)\end{bmatrix}\tilde{q}_y \qquad (37)$$

and the power spectral densities $\tilde{q}_x=\tilde{q}_y=6\,\text{m}^2/\text{s}^3$. The initial bias estimate of sensor i is zero with the initial bias covariance $\Sigma_i(0|0)=\text{diag}[(100\,\text{m})^2,(200\,\text{mrad})^2,(0.01)^2,(0.1)^2]$.

The simulation results are based on 100 Monte Carlo runs. The Normalized Estimation Error Squared (NEES) and the 95% probability interval of the proposed EXX method is shown in Figure 4 and it can be seen to be in its acceptance region for every step k, which means the estimator is consistent.

The RMS errors and the CRLB of the range offset biases are shown in Figure 5. The curves of the RMS errors are close to the curves of the CRLB with one

	Asynchronous Case		"Equivalent" Synchronous Case		ratio of
	RMS	σ of CRLB	RMS	σ of CRLB	$\sigma_{\text{asyn}}/\sigma_{\text{syn}}$
b_1^r	8.4 m	8.4 m	4.6 m	5.1 m	1.6
b_1^θ	0.17 mrad	0.18 mrad	9.9×10^{-2} mrad	0.11 mrad	1.6
ϵ_1^r	1.1×10^{-4}	1.1×10^{-4}	5.9×10^{-5}	6.8×10^{-5}	1.6
ϵ_1^θ	2.1×10^{-4}	2.2×10^{-4}	1.3×10^{-4}	1.3×10^{-5}	1.7
b_2^r	7.7 m	8.0 m	4.9 m	5.1 m	1.6
b_2^θ	0.52 mrad	0.51 mrad	0.32 mrad	0.34 mrad	1.5
ϵ_2^r	1.0×10^{-4}	1.1×10^{-4}	7.1×10^{-5}	6.8×10^{-5}	1.6
ϵ_2^θ	2.1×10^{-4}	2.0×10^{-4}	1.2×10^{-4}	1.3×10^{-4}	1.5

Table 1: Comparison of the Asynchronous (Different Rates) with Synchronous Scenario

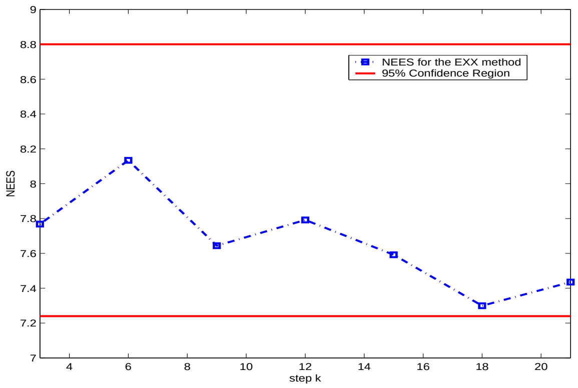

Fig. 4: NEES of the Biases

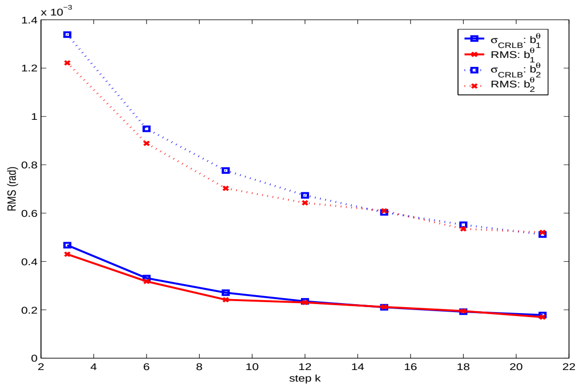

Fig. 6: RMS Errors and CRLB of the Azimuth Offset Biases

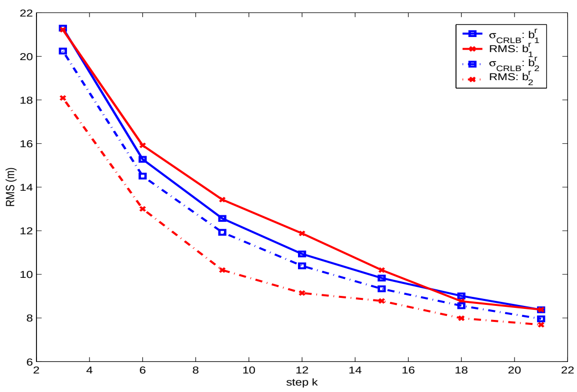

Fig. 5: RMS Errors and CRLB of the Range Offset Biases

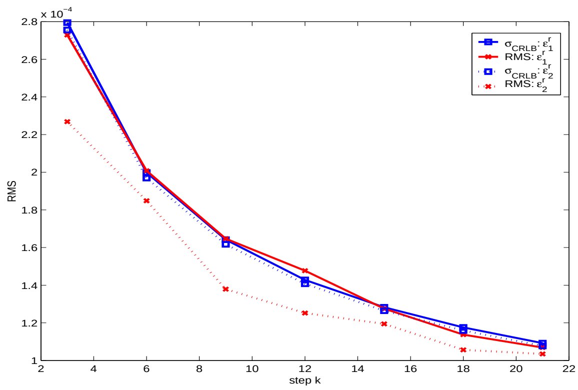

Fig. 7: RMS Errors and CRLB of the Range Scale Biases

RMS curve slightly above its CRLB and the other RMS curve slightly below its CRLB. Statistically, the RMS curves are in the confidence interval of their CRLB. The RMS errors and the standard deviations of the corresponding CRLB are listed on Table 1. The RMS error of the range offset bias for the first sensor b_1^r is 8.4 m at step 21, which is the same as the standard deviations of the corresponding CRLB. For sensor 1, the RMS error of the azimuth offset bias b_1^θ is 0.17 mrad, which is close to the standard deviation of its CRLB, which is 0.18 mrad. Similar trends can be found for the azimuth offset biases, the range and azimuth scale biases where the RMS errors are very close to their CRLB as shown in Figures 6–8.

Degradation Analysis

In this scenario (with the sensor sampling intervals of 1 s and 3 s, and the offset of 2.5 s), there are four measurements in a slot (n=4). We have the times in the first slot t_1=1 s, t_2 =2 s, t_3=3 s from sensor 1 and t_4=3.5 s from sensor 2. Then $\alpha_1(j)$ and $\alpha_2(j)$ can be obtained from (20) and (21). We have

$$\alpha_1(2) = -1.5 \quad \alpha_2(2) = 2.5 \tag{38}$$

$$\alpha_1(3) = -0.25 \quad \alpha_2(3) = 1.25 \tag{39}$$

Since the measurement noise will be the dominant factor in the pseudomeasurement noise of the biases, the variance of the pseudomeasurement noise can be approximated by ignoring the process noise covariance in

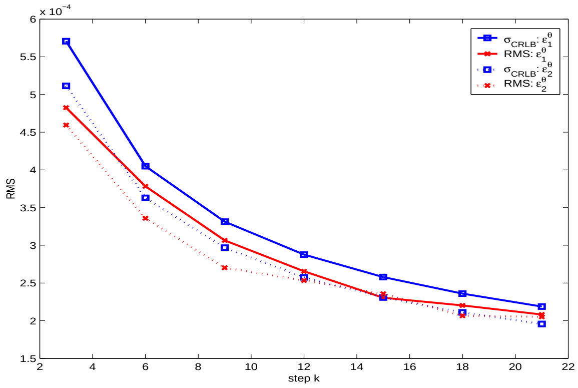

Fig. 8: RMS Errors and CRLB of the Azimuth Scale Biases

(32) and (33). Then we have

$$R_b(2,2) \approx R_0 + \alpha_1(2)^2 R_0 + \alpha_2(2)^2 R_0 \quad (40)$$
$$R_b(3,3) \approx R_0 + \alpha_1(3)^2 R_0 + \alpha_2(3)^2 R_0 \quad (41)$$
$$R_b(2,3) \approx R_0 + \alpha_1(2) R_0 \alpha_1(3) \quad (42)$$

In this scenario (4 measurements in a slot) with n observations from sensor 1 and $n/3$ observations from sensor 2 (a total of $4n/3$ observations, i.e., $n/3$ slots), the variance of the bias estimates is approximately

$$P_b^{\mathrm{asyn2}} \approx \frac{\left\{ \begin{bmatrix} \mathcal{H}' & \mathcal{H}' \end{bmatrix} R_b^{-1} \begin{bmatrix} \mathcal{H} \\ \mathcal{H} \end{bmatrix} \right\}^{-1}}{n/3} = \frac{7.3}{n} \left[\mathcal{H}' R_0^{-1} \mathcal{H} \right]^{-1} \quad (43)$$

For two synchronous sensors with the same total of $4n/3$ observations[4], the variance of the bias estimates is approximately

$$P_b^{\mathrm{syn2}} \approx \frac{\left[\mathcal{H}' (2R_0)^{-1} \mathcal{H} \right]^{-1}}{2n/3} = \frac{3}{n} \left[\mathcal{H}' R_0^{-1} \mathcal{H} \right]^{-1} \quad (44)$$

The ratio of the standard deviation of the bias estimates for the asynchronous sensors to that of the synchronous sensors is about $\sqrt{7.3/3} \approx 1.56$, i.e., a 56% increase. In our simulation, the ratio of the standard deviations of the CRLB for the range offset bias of sensor 1 is 1.6 as shown on Table 1. For other biases, the ratios are between 1.5 to 1.7, which are very close to our theoretical analysis.

5 Conclusions

In this paper we provide the exact solution for the multisensor bias estimation problem for general asynchronous sensors using common targets of opportunity. A novel scheme is proposed to transform the measurements from different times into *state-independent* pseudomeasurements of the biases such that the pseudomeasurement noises of the asynchronous sensor biases are zero-mean, white and with easily calculated covariances.

The main advantages and features of the proposed EXX method can be summarized as follows: the solution is exact, no approximation has been made except in the linearization of the nonlinear measurements; the algorithm is implemented recursively which is computationally efficient; these results enable the evaluation of the Cramer-Rao Lower Bound on the covariance of the bias estimates, i.e., the quantification of the available information about the biases. The proposed EXX method has also been shown to be statistically efficient, i.e., the CRLB is achievable. The magnitude of the CRLB indicates in the scenarios considered the information available from the observations does not allow a more accurate estimate.

[4] We assume the total of $4n/3$ observations are equally divided between the two sensors ($2n/3$ to each, thus $2n/3$ slots) in order to have an "equivalent" synchronous scenario.

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