On-line Bias Estimation of Secondary Radar Networks for ATC[1]

A. Soto Jaramillo, J. A. Besada Portas, J. R. Casar Corredera, J. I. Portillo García
Universidad Politécnica de Madrid
Despacho C-315.1. ETSI de Telecomunicación
c/ Ciudad universitaria s/n
28040 Madrid
Spain
andres@grpss.ssr.upm.es; besada@grpss.ssr.upm.es; jramon@grpss.ssr.upm.es; javierp@grpss.ssr.upm.es

Abstract – *Bias estimation is a prerequisite to data fusion in ATC environments. Radar networks and aircraft transponders have different bias terms which should be cancelled in order to perform a correct multisensor tracking. In this paper, several general architectures to overcome these problems will be proposed, taking into account both radar and aircraft biases. Its performance, both in terms of accuracy and in terms of computer load, will be analyzed.*

Keywords: Radar network, bias estimation, tracking, air traffic control.

1 Introduction

Modern air surveillance systems are composed of sensor networks with the aim of achieving better inferences than those coming from a single sensor.

These sensors are measure sources that must be combined in a data fusion process. This implies that data fusion needs a correct model of measurement errors, including random and systematic ones (biases).

In this situation, bias estimation is a prerequisite to data fusion in ATC environments. Radar networks and aircraft transponders [3] have different bias term which should be cancelled in order to perform a correct multisensor tracking.

In this article the multisensor bias problem in ATC environments is introduced. The impact of the existence of biases in the tracking process, several algorithms that could estimate these biases and finally, the performance improvement when these algorithms are included in the tracking process, are also presented.

The first section deals with the bias problem in environments with several radars. It defines the concept of bias, why they appear and explains the impact of not canceling them. The measure error model applied in this architecture is also presented.

Section 3 presents the general processing architecture, describing each of its parts. In section 4, the bias estimation method is developed. Section 5, presents how to generalize the bias estimation method to more than two radars.

Section 6 shows four possible combinations and temporal smoothing schemes for processing radar measures. Their features are analyzed allowing the election of one of them. Section 7 faces the problem of estimating transponder bias. It defines the problem and presents a possible estimation method. Finally, in Section 8, there are some simulation results, which show the improvement obtained using bias estimation methods. More about these topics could be found in [4,5].

2 Bias problem in Multiradar detection

As a bias we consider a systematic constant or slowly variant error in all measures. These biases are mainly due to:

- Incorrect sensor calibration
- Signal propagation

If these errors are not taken into account, a degradation of the multiradar tracking performance may be observed. That is the main reason why they should be cancelled from sensor measures before passing them to the rest of the processing system.

As we will develop in next sections, these biases depend not only on the radar but also on aircraft transponders.

The effect of biases is double:

- On one hand, they propagate the error to the final estimators, because the tracking filter is a low band filter so it does not cancel these almost constant errors. The most important consequence is that the tracking system can generate several tracks for the same target, one for each radar that detects it.
- On the other hand, it causes the appearance of certain instabilities in the tracks, giving more likelihood to the existence of manoeuvres than it should do. The reason is that plots arriving from each one of the radars can be quite far away from the ones arriving from the others. This causes the tracking filter supposes that the aircraft is following a trajectory in which it develops one manoeuvre after another.

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The measure error has usually been modelled as a white Gaussian zero-mean noise. We define a measure model which includes the presence of biases. A SSR measures target range and azimuth (R,θ). We are presenting the terms appearing in each of the measures due to the presence of both biases and noise terms. This analysis is going to focus on two dimensions for simplicity reasons, but it could be easily generalized for three dimensions, including height (R,θ, h).
The measure model for a SSR is:

$$R_m = (1 + K)R + \Delta R + b_{aircraft} + n_R \tag{1}$$

$$\theta_m = \theta + \Delta\theta + n_\theta \tag{2}$$

where:

- R_m: measured range
- R: actual range
- θ_m: measured azimuth
- θ: actual azimuth

Measure errors in range measures are divided into four components. A more detailed analysis of error sources either in range or in azimuth could be found in [1].

The fist one is ΔR, the *constant range bias*. It groups all the constant biases in the measure obtained from the radar. The next one is K which represents *the linear variation of biases with range*. This term is due to the propagation of the signal across the troposphere. Another term is $b_{aircraft}$ which represent the *aircraft transponder error*. This error represents the variation of the answering delay that the transponder waits to transmit the response to the SSR. Last term is the measure noise (n_R), which is characterized as a white Gaussian zero-mean noise.

In the azimuth measures, all errors are divided into two terms. One that groups all the biases ($\Delta\theta$), which is consider to be almost constant and a noise term (n_θ), are considered a white Gaussian zero-mean noise.

All the estimation methods that are going to be presented are based on processing differences of measurements taken from pairs of sensors and from the same aircraft. In next section, a general processing architecture is presented, showing how measures are going to be processed.

3 General bias estimation scheme

Tasks implemented in this approach are developed in two steps by two different blocks. The following figure presents the general processing architecture:

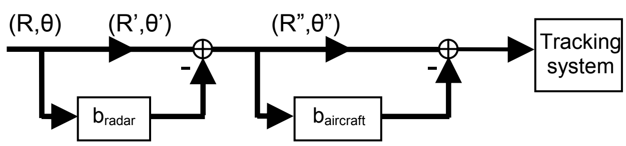

Fig. 1. Global biases estimation subsystem

In the first block, radar biases are calculated. The function implemented in this subsystem estimates the radar depending biases every T seconds/minutes, and removes them from radar measures during next T seconds/minutes, until next bias estimation is obtained. In this block, measures are accumulated until the moment in which the estimation method is applied, every T seconds/minutes.

The next step is associated with each individual target. In this second block the aim is to obtain the bias introduced by the aircraft transponder. This bias is related with the error in the response time to the SSR query that is present in the aircraft transponder. It is considered that this bias is seen by the radar bias estimator subsystem as a noise which adds to the measure one. This assumption can be made because in a normal ATC environment radars will detect several targets. The number of target is assumed large enough to consider that aircraft depending biases as white noise for this first estimator.

The estimation of this bias is calculated for every measure, which in this point should not have important radar biases, and cancelled afterwards. The estimation method will be detailed in sec. 7.

Finally, the "bias free" processed measure is then delivered to the tracking system an also to the rest of the system.

4 Radar bias estimation methods

In this section, several estimation methods for bias estimation are proposed. Each of these methods estimates radars biases (ΔR, K, Δθ).

The theoretic methods that are proposed are:

- Bias estimation based on a LSE estimator
- Bias estimation based on a MSE estimator
- Bias estimation based on a Kalman Filter

All these radar bias estimation methods are based on processing differences of measurements taken from pairs of sensors and referred to the same aircraft. It is important to notice that the first measure used in the measurement pair needs to be extrapolated (using the velocity estimation of the track to which the two plots are associated with) to the second measure timestamp.

Assuming the extrapolation introduces a negligible error, the difference of measurements in Cartesian coordinates can be modeled as:

$$\Delta x = f_{x2}(R_{m2}, \theta_{m2}) - f_{x1}(R_{m1}, \theta_{m1}) \tag{3}$$

$$\Delta y = f_{y2}(R_{m2}, \theta_{m2}) - f_{y1}(R_{m1}, \theta_{m1}) \tag{4}$$

Where $f_{x1}(\cdot)$, $f_{x2}(\cdot)$, $f_{y1}(\cdot)$ and $f_{y2}(\cdot)$ are coordinate transformation from Polar to Cartesian 'x' and 'y' components respectively and the numeric sub index refers to the appropriate radar.

Using the error model stated in Eqs. (1,2) it is possible to linearize Eqs. (3,4) around the ideal position with null biases and noises, obtaining:

$$f_{xi}((1+K_i)R_i + \Delta R_i + n_{Ri}, \theta_i + \Delta\theta_i + n_{\theta i}) \cong \tag{5}$$
$$f_{xi}(R_i, \theta_i) + K_i \frac{\partial f_{xi}}{\partial K_i} + \Delta R_i \frac{\partial f_{xi}}{\partial \Delta R_i} + \Delta\theta_i \frac{\partial f_{xi}}{\partial \Delta\theta_i} + n_{Ri} \frac{\partial f_{xi}}{\partial n_{Ri}} + n_{\theta i} \frac{\partial f_{xi}}{\partial n_{\theta i}}$$

$$f_{yi}\left((1+K_i)R_i+\Delta R_i+n_{Ri},\theta_i+\Delta\theta_i+n_{\theta i}\right)\cong \quad (6)$$

$$f_{yi}(R_i,\theta_i)+K_i\frac{\partial f_{yi}}{\partial K_i}+\Delta R_i\frac{\partial f_{yi}}{\partial \Delta R_i}+\Delta\theta_i\frac{\partial f_{yi}}{\partial \Delta\theta_i}+n_{Ri}\frac{\partial f_{yi}}{\partial n_{Ri}}+n_{\theta i}\frac{\partial f_{yi}}{\partial n_{\theta i}}$$

Where different radars are distinguished by the sub index 'i'. The differentiated positions from the pair of radars (1,2) translated to horizontal coordinates can be modeled as:

$$\Delta x\cong\left(K_2\frac{\partial f_{x2}}{\partial K_2}+\Delta R_2\frac{\partial f_{x2}}{\partial \Delta R_2}+\Delta\theta_2\frac{\partial f_{x2}}{\partial \Delta\theta_2}\right)-\left(K_1\frac{\partial f_{x1}}{\partial K_1}+\Delta R_1\frac{\partial f_{x1}}{\partial \Delta R_1}+\Delta\theta_1\frac{\partial f_{x1}}{\partial \Delta\theta_1}\right)$$
$$+\left(n_{R2}\frac{\partial f_{x2}}{\partial n_{R2}}+n_{\theta 2}\frac{\partial f_{x2}}{\partial n_{\theta 2}}-n_{R1}\frac{\partial f_{x1}}{\partial n_{R1}}-n_{\theta 1}\frac{\partial f_{x1}}{\partial n_{\theta 1}}\right) \quad (7)$$

$$\Delta y\cong\left(K_2\frac{\partial f_{y2}}{\partial K_2}+\Delta R_2\frac{\partial f_{y2}}{\partial \Delta R_2}+\Delta\theta_2\frac{\partial f_{y2}}{\partial \Delta\theta_2}\right)-\left(K_1\frac{\partial f_{y1}}{\partial K_1}+\Delta R_1\frac{\partial f_{y1}}{\partial \Delta R_1}+\Delta\theta_1\frac{\partial f_{y1}}{\partial \Delta\theta_1}\right)$$
$$+\left(n_{R2}\frac{\partial f_{y2}}{\partial n_{R2}}+n_{\theta 2}\frac{\partial f_{y2}}{\partial n_{\theta 2}}-n_{R1}\frac{\partial f_{y1}}{\partial n_{R1}}-n_{\theta 1}\frac{\partial f_{y1}}{\partial n_{\theta 1}}\right) \quad (8)$$

As could be seen, this linearized model relates the observed measurement differences (Δx, Δy), with the parameters of systematic errors for a pair of radars and also relates the random error in original measurements with its effect on transformed coordinates. This relationship is the one that allows us to obtain bias values.

For simplification, results are presented assuming Cartesian projection on a Flat-Earth model (disregarding effects of height errors). The equations of that transformation are:

$$D_i=\sqrt{\left((1+K_i)R_i+\Delta R_i+n_{Ri}\right)^2} \quad (9)$$

$$f_{xi}=D_i\cos(\theta_i+\Delta\theta_i+n_{\theta i})+X_{Ri} \quad (10)$$

$$f_{yi}=D_i\sin(\theta_i+\Delta\theta_i+n_{\theta i})+Y_{Ri} \quad (11)$$

being (X_{Ri},Y_{Ri}) the position of i-th radar on Cartesian plane. So the elements in the model can be approximated as follows:

$$\frac{\partial f_{xi}}{\partial \Delta R_i}=\frac{R_{mi}}{D_i}\cos\theta_{mi}; \quad (12) \qquad \frac{\partial f_{yi}}{\partial \Delta R_i}=\frac{R_{mi}}{D_i}\sin\theta_{mi} \quad (17)$$

$$\frac{\partial f_{xi}}{\partial K_i}\cong\frac{R^2_{mi}}{D_i}\cos\theta_{mi}; \quad (13) \qquad \frac{\partial f_{yi}}{\partial K_i}\cong\frac{R^2_{mi}}{D_i}\sin\theta_{mi} \quad (18)$$

$$\frac{\partial f_{xi}}{\partial \Delta\theta_i}=-D_i\sin\theta_{mi}; \quad (14) \qquad \frac{\partial f_{yi}}{\partial \Delta\theta_i}=D_i\cos\theta_{mi} \quad (19)$$

$$\frac{\partial f_{xi}}{\partial n_{Ri}}=\frac{R_{mi}}{D_i}\cos\theta_{mi}; \quad (15) \qquad \frac{\partial f_{xi}}{\partial n_{Ri}}=\frac{R_{mi}}{D_i}\sin\theta_{mi}; \quad (20)$$

$$\frac{\partial f_{xi}}{\partial n_{\theta i}}=-D_i\sin\theta_{mi}; \quad (16) \qquad \frac{\partial f_{yi}}{\partial n_{\theta i}}=D_i\cos\theta_{mi} \quad (21)$$

The algorithms detailed could be classified in block methods and recursive methods.

4.1 Bias estimation based on a LSE estimator

From the linearized model of the previous section is found that there are six independent variables to be estimated. This implies at least that six independent measures are needed for obtaining three differences of measurements. Presenting the previous model in matrix form:

$$\begin{bmatrix}\Delta x_j\\ \Delta y_j\end{bmatrix}=\begin{bmatrix}\frac{\partial f_{x2,j}}{\partial K_2} & \frac{\partial f_{x2,j}}{\partial \Delta R_2} & \frac{\partial f_{x2,j}}{\partial \Delta\theta_2} & -\frac{\partial f_{x1,j}}{\partial K_1} & -\frac{\partial f_{x1,j}}{\partial \Delta R_1} & -\frac{\partial f_{x1,j}}{\partial \Delta\theta_1}\\ \frac{\partial f_{y2,j}}{\partial K_2} & \frac{\partial f_{y2,j}}{\partial \Delta R_2} & \frac{\partial f_{y2,j}}{\partial \Delta\theta_2} & -\frac{\partial f_{y1,j}}{\partial K_1} & -\frac{\partial f_{y1,j}}{\partial \Delta R_1} & -\frac{\partial f_{x1,j}}{\partial \Delta\theta_1}\end{bmatrix}\begin{bmatrix}K_2\\ \Delta R_2\\ \Delta\theta_2\\ K_1\\ \Delta R_1\\ \Delta\theta_1\end{bmatrix}+\varepsilon_j \quad (22)$$

where the index j is the index of the measurement pair, while ε_j is the adjustment error for j-th measure. The partial derivatives included in the projection matrix should be evaluated in the ideal point, but as this is unknown, it will be evaluated in the measure point. The later expression can be reduced to:

$$\vec{x}_j=H_j\vec{r}+\varepsilon_j \quad (23)$$

where $\vec{x}_j$ is the measurement difference (Δx, Δy) associated with the j-th pair of measures, H_j is the projection matrix, while $\vec{r}$ is a vector that contains the six parameters to estimate.

The least square solution is the one that for certain parameter values has the lowest total adjustment-error power. In other words, it minimized:

$$J=\sum_{j=1}^{N}\varepsilon_j^T\varepsilon_j \quad (24)$$

Next, measure vector $\vec{x}$ is defined as a column vector that groups all measures obtained in a certain period of time. This vector could be modeled as:

$$\vec{x}=\begin{bmatrix}\vec{x}_1\\ \vec{x}_2\\ \vdots\\ \vec{x}_N\end{bmatrix}=\begin{bmatrix}H_1\\ H_2\\ \vdots\\ H_N\end{bmatrix}\vec{r}+\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \vdots\\ \varepsilon_N\end{bmatrix}=H\vec{r}+\varepsilon \quad (25)$$

Finally, the LSE estimator is:

$$\hat{r}=H^{\#}\vec{x}=\left(H^TH\right)^{-1}H^T\vec{x} \quad (26)$$

This expression could be written in function of each measure difference, as follows:

$$\hat{r}=\left(\sum_{j=1}^{N}H_j^TH_j\right)^{-1}\sum_{j=1}^{N}H_j^T\vec{x}_j \quad (27)$$

For resolving this equation it is necessary at least tree linear independent measure differences.

4.2 Bias estimation based on a MSE estimator

In this method, measures can be modeled as in the previous subsection, Eq. 22, with only one change, consisting in interpreting ε_i as the error associated with the measures. Defining ε_i as:

$$\varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_N \end{bmatrix} \quad (28)$$

It could be modeled as a Gaussian, zero-mean random vector with covariance matrix R. So it could be proved that the MMSE estimator [2] has the form:

$$\hat{r} = \left(H^T R^{-1} H\right)^{-1} H^T R^{-1} \vec{x} \quad (29)$$

where H is the projection matrix presented in Eq. 25.

The main difference with previous method is calculating the covariance matrix R. From the development presented at the beginning of Sec. 4, especially in Eqs. (5-8), the measure error vector (projected over Cartesian coordinates) could be approximated as:

$$n_j = \begin{bmatrix} n_{x,j} \\ n_{y,j} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_{x2,j}}{\partial n_{R2}} & \frac{\partial f_{x2,j}}{\partial n_{\theta 2}} & -\frac{\partial f_{x1,j}}{\partial n_{R1}} & -\frac{\partial f_{x1,j}}{\partial n_{\theta 1}} \\ \frac{\partial f_{y2,j}}{\partial n_{R2}} & \frac{\partial f_{y2,j}}{\partial n_{\theta 2}} & -\frac{\partial f_{y1,j}}{\partial n_{R1}} & -\frac{\partial f_{y1,j}}{\partial n_{\theta 1}} \end{bmatrix} \begin{bmatrix} n_{R2,j} \\ n_{\theta 2,j} \\ n_{R1,j} \\ n_{\theta 1,j} \end{bmatrix} = F_j n_{0,j} \quad (30)$$

where F_j is the error projection matrix and $n_{0,j}$ is a vector containing the noise in the measure coordinates. This way, the covariance matrix of n_j can be obtained as (assuming zero-mean):

$$R_i = E\{n_j n_j^T\} = F_j E\{n_{0,j} n_{0,j}^T\} F_j^T = \\ = F_j \begin{bmatrix} \sigma_{R2}^2 & 0 & 0 & 0 \\ 0 & \sigma_{\theta 2}^2 & 0 & 0 \\ 0 & 0 & \sigma_{R1}^2 & 0 \\ 0 & 0 & 0 & \sigma_{\theta 1}^2 \end{bmatrix} F_j^T \quad (31)$$

If we assume that every measurement difference is independent, which is true if measurement noises from each radar can be considered white noise, the covariance matrix of the differences R could be expressed as:

$$R = \begin{bmatrix} R_1 & 0 & \cdots & 0 \\ 0 & R_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_N \end{bmatrix} \quad (32)$$

The form of matrix R allows us to modify the functional expression of the estimator, reaching another expression which requires fewer operations. It can be seen that:

$$R^{-1} = \begin{bmatrix} R_1^{-1} & 0 & \cdots & 0 \\ 0 & R_2^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_N^{-1} \end{bmatrix} \quad (33)$$

Introducing (33) in the estimator's expression leads to a simplified one, as is developed below:

$$\hat{r} = \left(H^T R^{-1} H\right)^{-1} H^T R^{-1} \vec{x} =$$

$$= \left(\begin{bmatrix} H_1^T & H_2^T & \cdots & H_N^T \end{bmatrix} \begin{bmatrix} R_1^{-1} & 0 & \cdots & 0 \\ 0 & R_2^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_N^{-1} \end{bmatrix} \begin{pmatrix} H_1 \\ H_2 \\ \vdots \\ H_N \end{pmatrix} \right)^{-1} H^T R^{-1} \vec{x} =$$

$$= \left(\sum_{j=1}^{N} H_j^T R_j^{-1} H_j \right)^{-1} \begin{bmatrix} H_1^T & H_2^T & \cdots & H_N^T \end{bmatrix} \begin{bmatrix} R_1^{-1} & 0 & \cdots & 0 \\ 0 & R_2^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_N^{-1} \end{bmatrix} \begin{bmatrix} \vec{x}_1 \\ \vec{x}_2 \\ \vdots \\ \vec{x}_N \end{bmatrix} =$$

$$= \left(\sum_{j=1}^{N} H_j^T R_j^{-1} H_j \right)^{-1} \sum_{j=1}^{N} H_j^T R_j^{-1} \vec{x}_j \quad (34)$$

4.3 Bias estimation based on a Kalman Filter

Another possibility is to model the problem as a dynamic one, so an extended Kalman Filter (EKF) can be applied. In principle, we will have six state variables and every difference should be taken as a measure for the filter. The models expressed in a Kalman Filter form will be:

4.3.1 Prediction model

$$\vec{r}_k = F \vec{r}_{k-1} + \nu_k = I_{6x6} \vec{r}_{k-1} \quad (35)$$

It is assume that the prediction matrix is the identity and there is no plant noise.

4.3.2 Measurement model

$$\vec{x}_k = H_k \vec{r}_k + n_k \quad (36)$$

All the terms in this expression have been already defined in previous sections.

The Kalman filter associated is defined as a recursive one, with the following equations:

$$G_k = P_{f,k-1} H_k^T \left(H_k P_{f,k-1} H_k^T + R_k \right)^{-1} \quad (37)$$

$$P_{f,k} = \left(I_{6x6} - G_k H_k \right) P_{f,k-1} \quad (38)$$

$$\hat{r}_{f,k} = \hat{r}_{f,k-1} + G_k \left(\bar{x}_k + H_k \hat{r}_{f,k-1} \right) \quad (39)$$

where G_k is the Kalman filter gain for the k-th sample. $P_{f,k}$ is the covariance matrix of the filtered estimator which is estimated by the filter as the k-th measure is available. R_k has been defined in Eq. (31).

Simulations of all these methods have proved that although all of them behave relatively well, it could be said that the better ones are MSE estimator and the one based on a Kalman Filter. Estimations obtained with these two methods are equal if the noise plant covariance matrix is zero, although MSE estimator is computationally more efficient.

Another thing observed was that although all methods behave well, neither of them reached the real biases values. The reason for this bias in the estimation was that we were using a linear approximation of the transformation functions, Eqs. (5-6); so at the time the estimated biases were bigger, the bias in the estimation also increased. The solution was to change the estimation target. We change from estimating total biases to estimating differential increases from the previous time interval estimate. This change eliminates the bias in the estimation as we are moving nearer to the point where the transformation function has been linearly approximated.

After obtaining bias estimation, there are at least two possibilities. The first one is to directly deliver this estimation to the rest of the system for removing the biases present in the measures. It could be seen that this estimation is independent from the previous ones. The second possibility is to combine the new estimation with the previous ones.

If we only keep last estimation, we will be loosing information, which means estimation degradation. So it is interesting to introduce a combination method which adapts to the quality of the estimations. This method should forget partially the oldest estimation and calculate what weights should be applied to the remaining passed estimations and to the new one. All these weights depend on the quality of both estimators. There are several possible implementations for this method. One of them is to modify measure block size, depending on if the estimation reaches a certain quality level. Another possible implementation is to introduce an extended Kalman filter. This option will be chosen for implementing this estimation time filtering.

As it is mentioned, bias estimation depends on aircraft traffic. This situation can be improved if we integrate in the bias estimation measures from fixed transponders. If these measures are processed, there are several considerations to bear in mind. The first one is that if only one transponder is used, it is impossible to discriminate between ΔR and K. Another one is that these measures must be introduced with care because they can introduce biases in the estimation. There are two possible schemes to introduce that kind of measures:

- Centralized: it processes those measures as if they were measurement differences.
- Distributed: it processes those measures separately from the measurement differences. This scheme needs a combination method to obtain the estimation from that kind of measures.

The centralized scheme is better, if the model is totally consistent with measures, because it combines measures in an optimal way. The problem is that if there are model inconsistencies, this scheme leads to a less robust solution. On the other hand, the distributed scheme needs fewer operations.

5 Generalization to more than two radars

In this section, the objective is to estimate in parallel biases of all radars placed in a certain area. There are several possibilities to generalize the system to more than two radars, some of them are presented below:

- Grouping radars in pairs. In this option each radar belongs only to a pair. For obtaining these pairs it should be kept in mind that the coverage of these two radars must be very overlapped. So it could be possible to obtain differences of measurement of almost all the coverage of each one of the radars. This method need N/2 filters working in parallel or (N+1)/2 if the number or radars is odd. The output of every filter, if there are no estimations that come from fixed transponders, will directly be the bias estimation that should be cancelled from measures. If there are estimations coming from fixed transponders, they should be combined with the ones coming from the aircraft traffic.
- Selecting for each radar an adjacent one. In this case, measure difference pairs coming from these two radars are only used to estimate the biases of the first one. This supposes N filters working in parallel. This method has several problems. The first one is that if the coverage of both radars is not very overlapped the number of measure pairs will not be big enough for obtaining a good estimation. Another problem is that not all the geometric combinations should be used. Restricting the estimation to a radar and another adjacent, can be misusing the improvement of using it with another radar. In certain situations there could be duplicated filters, so a filter can be saved. Therefore the number of filters should be between N/2 and N.
- Combinations of all the available estimations coming from all radars pairs. With N radars one can form N(N+1)/2 pairs. Each radar should be in N-1 of them, so at least can be N-1 possible bias estimations for each radar. Some of them are rejected because the coverage of the two radars is not overlapped enough. From the rest a combination is needed. One of the possible combinations is obtaining the mean value of the group of bias estimations and another is applying an optimal Bayesian combination.

6 Several schemes for measure combination and time smoothing

In the previous section, methods for combining several bias estimations obtained in different time instants and other methods for combining bias estimation obtained at the same time were described. But the combination of these two estimation processes can admit two possible orders:

- First the time smoothing for each radars pair, and after that, the combination of all the estimations for one radar.
- The combination of all the bias estimations for certain radar, followed by a time filtering of the bias estimation of each radar.

In these schemes, the same measure will be used to calculate biases in all radars pairs that are detecting that aircraft. This supposes a certain correlation between the estimation errors present in the estimations.

It is important that bias estimations were performed for synchronized time blocks for all the different sensors pairs.

In the following schemes several possible options are depicted. In these schemes, we assume that differential biases ($\partial x_i(k)$) are calculated, for each radar, after completing a measure block. $\Delta x(k)$ represents the final bias estimator (total biases) obtained after processing measures contained in k-th time interval. These values will be used to correct arriving measures in k+1 interval.

In the first method (Fig.2), we add total biases obtained in last interval to the difference biases obtained in this time interval. Previously, all the differential bias estimators related to a radar have been selected. Afterwards, a bias combination is performed, and finally a time smoothing, weighting previous biases with the new ones.

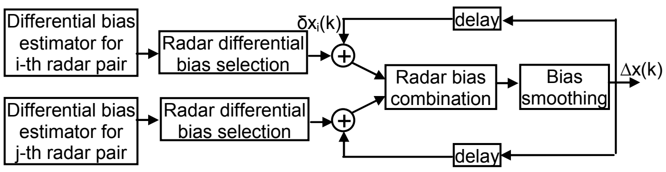

Fig. 2. Combination followed by time filtering

The second option (Fig.3) is very similar to the latest one. The main difference is that in this case a combination of differential corrections is performed, instead a combination of total corrections.

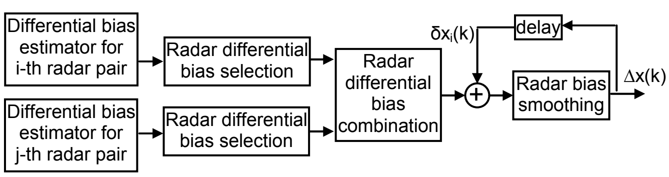

Fig. 3. Combination of differential corrections followed by time filtering

The third method (Fig.4) starts performing a time filtering of complete biases (after selecting and adding the differential estimators to the previous estimated ones) which correspond to each of the radar pairs and continues making a combination of these biases. The result is the complete bias estimator for each radar.

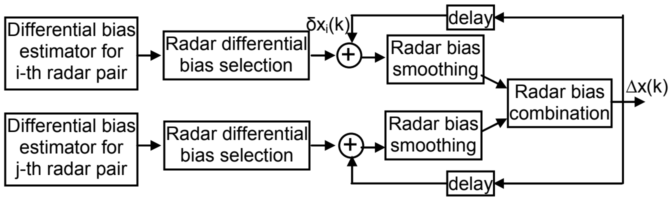

Fig. 4. Uncoupled time filtering followed by bias combination

The forth method (Fig.5) is a solution developed from the previous one. Here, instead of filtering all selected biases for certain radar in a separated way, biases belonging to the two radars are filtered together. Before doing that, previous estimations should be added to the differential ones, as could be seen in the figure.

The main difference with the previous method is that this one can extract information kept in cross correlations that the other one cannot. Nevertheless, this method has more computational load.

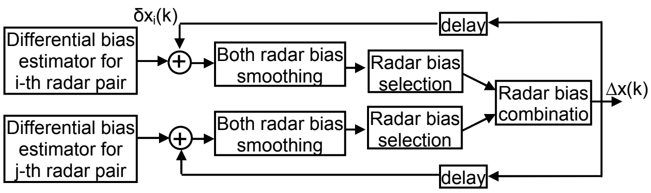

Fig. 5. Coupled time filtering followed by bias combination

Analyzing these four methods, it is seen that the first method is totally equivalent to the second one, which also has less computational load, so it can be rejected.

The second method is the one with lowest computation load, because it only has one time filter for radar, instead of one for each radar pair, as in the other methods. Considering that the estimation is only calculated once every T seconds/minutes, it could be seen that with a modern computer it does not take much time. So the load criterion is not very important.

Analyzing the consistency of the obtained estimators shows that as all measures are introduced to all radar pairs, either cross correlations are calculated or the estimations obtained are suboptimal.

The last two methods allow a consistent time filtering, even though the combination will be suboptimal. Since, these two methods use less suboptimal approximations, they are preferred. Between these two methods, the chosen is the latest one, because it makes use of the cross correlations between the estimators of the two radars in the time filter.

7 Estimating airborne transponder bias

As was mentioned in Sec. 2, the biases cancellation is divided into two parts, one depending on radars and another depending on aircraft transponders. In this section the estimation of airborne transponder bias is going to be presented.

This estimation must be obtained for each measure so accumulations of measures are not possible. Data received by this subsystem is assumed to have null radar bias. The measure model for a SSR which has null radar biases is:

$$R_m = R + b_{aircraft} + n_R \qquad (40)$$

$$\theta_m = \theta + n_\theta \qquad (41)$$

The measurements in Cartesian coordinates can be modeled as:

$$x_m = x + b_{aircraft} \cos\theta_m + n_x \qquad (42)$$

$$y_m = y + b_{aircraft} \sin\theta_m + n_y \qquad (43)$$

The differentiated positions from a pair of radars translated to horizontal coordinates can be modeled as:

$$\Delta\vec{r} = \begin{bmatrix} x_{mj} \\ y_{mj} \end{bmatrix} - \begin{bmatrix} x_{mi} \\ y_{mi} \end{bmatrix} = \begin{bmatrix} \cos\theta_j - \cos\theta_j \\ \sin\theta_j - \sin\theta_j \end{bmatrix} b_{aircraft} + \begin{bmatrix} n'_x \\ n'_y \end{bmatrix} \qquad (44)$$

This could be represented in matrix form as:

$$\Delta\vec{r} = \begin{bmatrix} \Delta x_{ji} \\ \Delta y_{ji} \end{bmatrix} = H\, b_{aircraft} + n \qquad (45)$$

In this form, it is easy to obtain the expression of an estimator using the same ideas in Sec. 4. Vector n is considered as a white zero-mean Gaussian noise whose covariance may be easily modeled. As in the radar biases case, there are several possible estimators. For this case, it has been chosen an estimator based on an extended Kalman filter, whose equations are very similar to the ones presented in Eqs. (35-39)

8 Simulation results and conclusions

The results are obtained from a simulated ATC scenario with ten aircraft uniformly distributed in the space and 2 radars to fuse, which are separated 200 Km, have a period of 4 s/scan and the following biases: ΔR=100m, K=150/max range, Δθ=0.04º.

From all the possible combinations for estimating radar biases, the MSE estimator is the chosen one. The results are presented in the figures below. In all the figures different biases are presented versus the number of measures; the line in red is the estimated value and the lines in blue represent the estimated value +/- σ (estimation covariance).

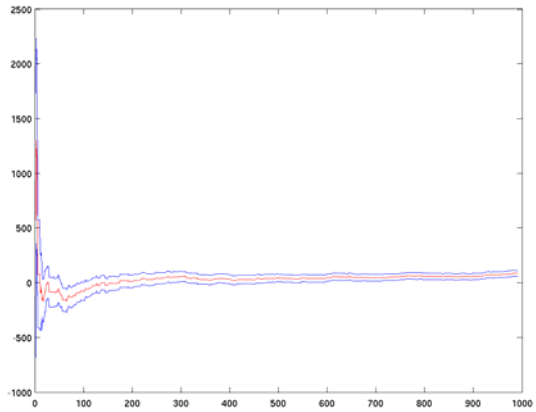

Fig. 6. Range bias (ΔR) estimation of radar 1

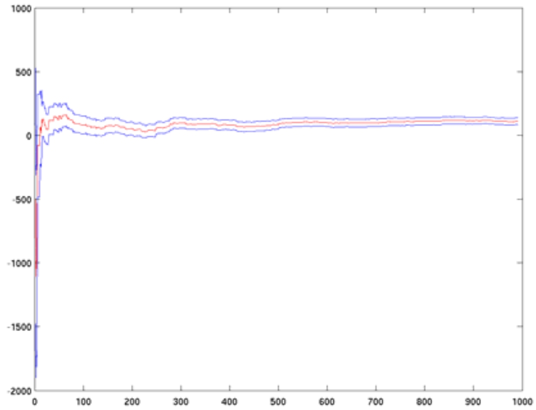

Fig. 7. Range bias (ΔR) estimation of radar 2

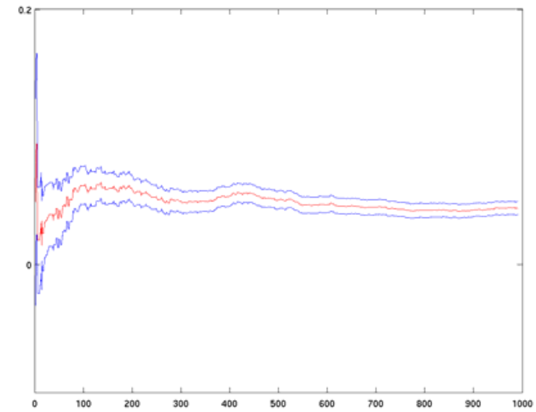

Fig. 8. Azimuth bias (Δθ) estimation of radar 1

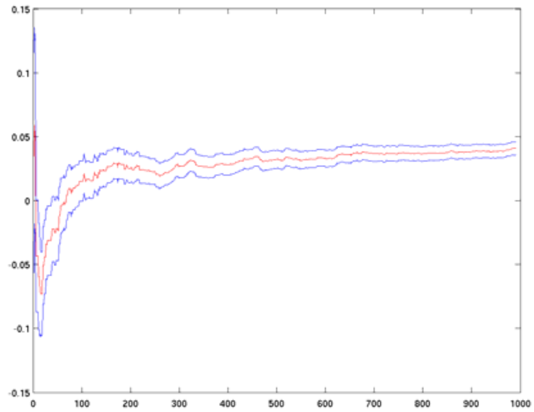

Fig. 9. Azimuth bias ($\Delta\theta$) estimation of radar 2

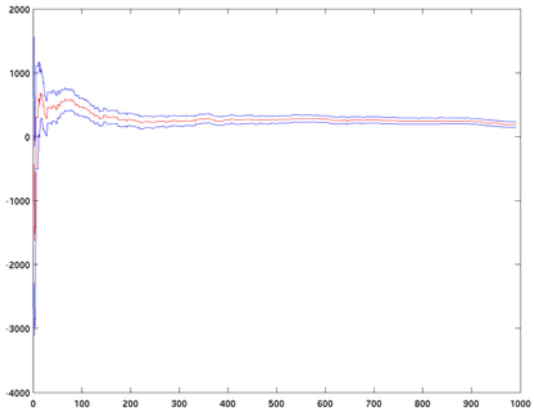

Fig. 10. Propagation bias (K) estimation of radar 1

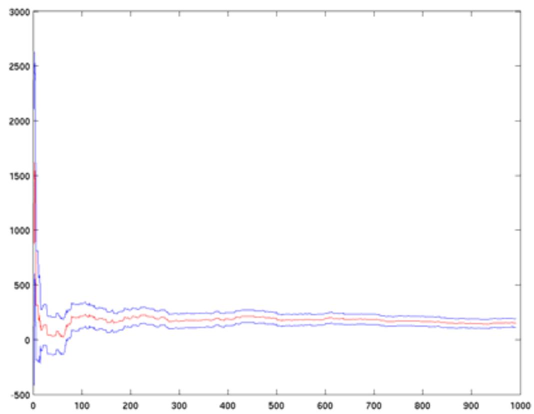

Fig. 11. Propagation bias (K) estimation of radar 2

In the case of the aircraft transponder bias (Fig. 12), the trajectory of the target analyzed crosses the line that links the two radars. The real value of the transponder bias is 50m. As in the other figures, the line in red is the estimated value and the lines in blue represent the estimated value +/- σ (estimation covariance).

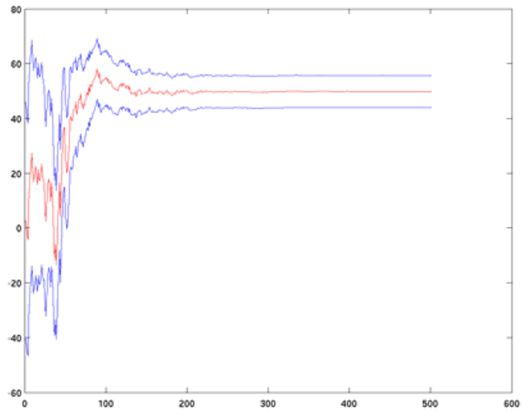

Fig. 12. Target transponder bias ($b_{aircraft}$)

In conclusion, as it is shown all the estimators converge to the real bias values. Another issue to consider is that final bias errors depend strongly on the scenario geometry defined by aircraft trajectories with respect to sensor positions. The higher variation of geometry parameters (number of aircraft, trajectory diversity and length, etc.) along the processed measures, the higher observability and accuracy to derive bias estimators.

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