Fusion of detections in a multi-carrier GPS receiver

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Abstract – *The work presented in this paper deals with the code tracking of the GPS signal. The code tracking is based on the shift code detection, where the shifts are provoked by the movement of the receiver and the satellite. We propose to fuse the shift code detection achieved on multi carrier frequencies. In this work we defined a MAP detection criterion of the shifts instants. This hybrid method of detection combines a centralized fusion criterion of detection with a decentralized one. The thresholds of decision associated to the fusion system are defined in the Neyman Pearson sense. The goal of the proposed method is to fused the shifts detections when they are not simultaneous like in this application. Indeed in reality there is a difference between the instants of shifts on the carrier frequencies L1 and L2, due to the effect of ionospheric propagation. The experimentations achieved on synthetic GPS signals show the interest of the method in relation to the classical algorithms.*

Keywords: GPS, information fusion, tracking, detection.

1 Introduction

The satellite navigation GPS is a three dimensional positionning system. It is based on the measurements of the distances between the receiver and a set of satellites. Each satellite is transmitting its own position in a navigation message and the receiver measures the transmission time from the satellites to the receiver. Actually, in a civil GPS receiver only one frequency L1 (1575.42 MHz), is used for broadcasting navigation data and ranging codes from the satellites, by the use of code division multiple access (CDMA). Each satellite uses a different ranging code. The navigation data provides the receiver the satellite location at the time of transmission. The ranging code may be used by the receiver to determine the propagation delay of the satellite signals. Then we can calculate the pseudo-distances between the receiver and a satellite with the speed of satellite signals (3.10^8m/s). If we suppose that the satellite and the receiver are not synchronized, we can calculate the receiver position and synchronization (x, y, z ,Δt) with four different pseudo-distances. [1].

Today the evolution of the GPS system passes by the increase of the number of carrier frequencies (evolution of NAVSTAR, GALILEO). For example in the futur NAVSTAR GPS system, a second civil signal will use the C/A code currently used at GPS L1 and will be located at GPS L2 (1227.6 MHz). The third civil signal, which is intended to meet the needs of critical safety-of-life applications, will be located at 1176.45 MHz (L5) [2].

The pseudo distance is measured with the propagation delay of the signal GPS between the satellite and the receiver. The receiver correlates the ranging code it received with a local code it generates. We can then deduce the visible satellites (each satellite has a different ranging code: GOLD code) and the delay of propagation from the maximum of the correlation. After a phase of acquisition where the local and received codes are synchronized, the receiver tracks the shifting of the local code provoked by the movements of the receiver and satellite. The tracking system in the receiver is confronted with the decision to shift or not the local code. This decision is taken with the noisy measures of the correlation calculated on the real GPS signal. In the receiver we found a tracking system for each satellite associated to its ranging code[1] [3]. In the future evolution of the GPS there will have several carrier frequencies, then it will be possible to have several tracking system working simultaneously for a same satellite. We present in this article a tracking fusion method applied to the future signal GPS multi carrier frequencies.

In the case of fusion we can define two detection approaches : the centralized detection and the distributed one. The distributed detection, very largely studied by [4] and [5], considers the detection at the level of each sensors and then carries out a global decision by combination (fusion) of the local decisions. The difficulties in this case lie in the definition of the thresholds at the level of each local detector. Most of the works on this subject are based on criterion to be optimized such as Bayes or Neyman -Pearson. In recent works the problem of the design of fusion rule is solved in an attractive way using a hierarchical model[6]. In this approach, the definition of a specific prior on each hypothesis is not necessary but the computational complexity of the method is a brake to its use in our application. Actually a great number of papers deals with the problem of correlated decision in the fusion case. In [7] an adaptative fusion algorithm is proposed to estimate prior and conditional probabilities. In [8], a Bayes-optimal binary quantization is presented and a Neyman-Pearson optimum distributed dectection shemes is proposed in [9].

The centralized detection system considers all the measurements to perform the decision. This system offers the best performances but the quantity of informations to be processed by the fusion system can quickly become significant. One of the drawbacks of this approach is its sensitivity to the synchronisation of the data.

The goal of this work is to fuse the detection realized on each tracking system performed on the carrier frequencies. We propose to fuse the shift code detection achieved on multi carrier frequencies to increase the robustness of detection in the noise presence. In this work we defined a MAP detection criterion of the shifts instants. This hybrid method of detection combines a centralized fusion criterion of detection with a decentralized one. The thresholds of decision associated to the fusion system are defined in the Neyman Pearson sense. The interest of the proposed method is that we can fuse the shifts detections when they are not simultaneous like in our application. Indeed the travel of the GPS signal through the ionosphere causes a group velocity delay of the waves [1]. This will cause no simultaneous codes shifts according to the frequencies of the carrier when the receiver and the satellite are moving.

The paper is organized as follow. Section 2 describes the GPS signal model. The MAP decision criterion is described in section 3 and the fusion method in section 4. In section 5 we present numerical experimentations on synthetic GPS signals.

2 Model of GPS signal

Let consider the expression of the in-phase and quadrature components after correlation and demodulation for each time of samples t_k [10] :

$$I_k = \sqrt{2C/N_0T}R_f(\tau_k)\cos(\phi_k) + n_{ik} \quad (1)$$

$$Q_k = \sqrt{2C/N_0T}R_f(\tau_k)\sin(\phi_k) + n_{qk} \quad (2)$$

With :

T = predetection bandwith where the correlation is done,
ϕ_k = residual phase tracking error at time t_k,
n_k = the in-phase and quadraphase noise samples,
R_f = correlation between filtered signal and the non-filtered code generated,
C/N_0 = signal-to-noise ratio normalized to a 1 Hz bandwith.

In the non-coherent case, the mean of the early-minus-late discriminator is given by :

$$E\left[D_{\tau_k}\right] = \bar{I}_E^2 + \bar{Q}_E^2 - \bar{I}_L^2 - \bar{Q}_L^2 \quad (3)$$

Where I_E and Q_E are the in-phase and quadrature component, correlated with a code which is generated slightly early. I_L and Q_L are the same components correlated with a code slightly late.This discriminator will be used in the experimentation on signal GPS.

We can notice here that we have the difference of chi-square variables with two degrees of freedom.

So for the mean :

$$E\left[D_{\tau_k}\right] = 2C/N_0T\left[R_f^2(\tau_k - \frac{T_c}{2}) - R_f^2(\tau_k + \frac{T_c}{2})\right] \quad (4)$$

And the variance :

$$\sigma^2_{D_{\tau_k}} = 8 + 8\,C/N_0T\left[R_f^2(\tau_k - \frac{T_c}{2}) + R_f^2(\tau_k + \frac{T_c}{2})\right] \quad (5)$$

When the position of the correlation maximum changes, the received code is shifted compared to the generated local code. Then a change of mean and variance occurs on the measures of the discriminator.
For illustration, we represent figure 1 the evolution of the values of I_E, I_L and I_P (Prompt) obtained on real data for a fixed receiver (moving satellite). In this representation, a shift of code occurs sample 25. The evolution of various discriminator is represented figure 2.

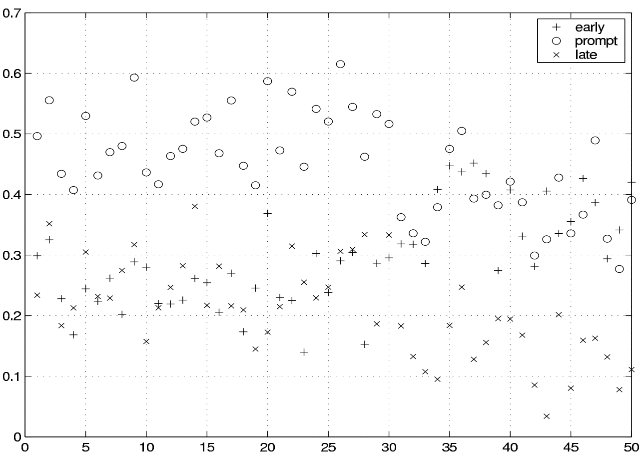

Fig. 1: Evolution of IE, IL and IP components.

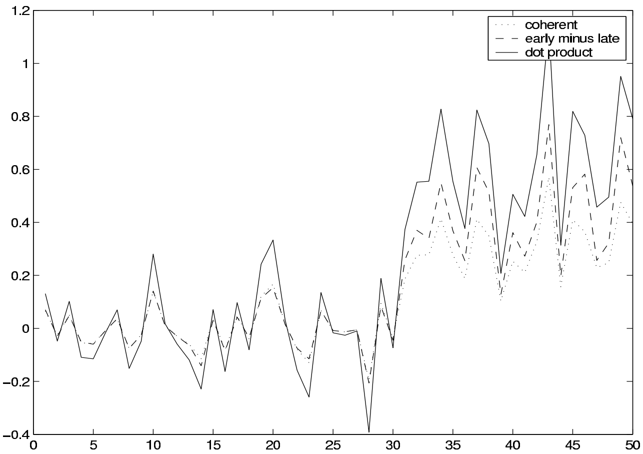

Fig. 2: Evolution of discriminators.

3 MAP decision criterion

The posterior probability to have a sequence of ruptures $\underline{r}$ in a signal $\underline{y}$ is written [11] :

$$\sup_{\{\underline{r},\,\underline{\theta}\}} P_r(\underline{R}=\underline{r}/\underline{Y}=\underline{y};\underline{\theta}) \tag{6}$$

We consider two hypothesis,H_1 and H_0 respectively for the presence and the absence of changes on a signal. We can then write the following rule of decision :

$$\begin{gathered}\sup_{\{\underline{r}^1\}} P_r(\underline{R}^1=\underline{r}^1/\underline{Y}=\underline{y};\underline{\theta}^1) \\ \lessgtr^{H_0}_{H_1} \\ \sup_{\{\underline{r}^0\}} P_r(\underline{R}^0=\underline{r}^0/\underline{Y}=\underline{y};\underline{\theta}^0)\end{gathered} \tag{7}$$

with :

$\underline{r}^1$ et $\underline{r}^0$, the changes sequences respectively associated to the H_1 and H_0 hypothesis,
$\underline{\theta}^1$ et $\underline{\theta}^0$, parameters corresponding respectively to the sequences $\underline{r}^1$ and $\underline{r}^0$ of presumedly known ruptures.

By using the rule of Bayes, we can write :

$$\begin{gathered}\sup_{\{\underline{r}^1\}} \{h\,(\underline{y}/\underline{r}^1;\underline{\theta}^1)\, f(\underline{\theta}^1)\, \pi(\underline{r}^1)\} \\ \lessgtr^{H_0}_{H_1} \\ \sup_{\{\underline{r}^0\}} \{h\,(\underline{y}/\underline{r}^0;\underline{\theta}^0)\, f(\underline{\theta}^0)\, \pi(\underline{r}^0)\}\end{gathered} \tag{8}$$

with :

$\pi(\underline{r}^i)$ = the prior density of the configurations of changes $\underline{r}^i$ associated to the hypothesis H_i, $i \in \{0,1\}$,
$f(\underline{\theta})$ = the prior density of the parameters supposed to be a uniform law on all the values of θ.

By taking the logarithm of the preceding expression, we can then write the decision criterion :

$$\begin{gathered}\sup_{\{\underline{r}^1\}} \{\ln(h\,(\underline{y}/\underline{r}^1;\underline{\theta}^1)) + \ln(\pi(\underline{r}^1))\} \\ \lessgtr^{H_0}_{H_1} \\ \sup_{\{\underline{r}^0\}} \{\ln(h\,(\underline{y}/\underline{r}^0;\underline{\theta}^0)) + \ln(\pi(\underline{r}^0))\}\end{gathered} \tag{9}$$

If we suppose $\underline{r}^i$ as a sequence of independent Bernouilli variables, we have :

$$\begin{gathered}\pi(\underline{r}^1) = \beta(1-\beta)^{n-1} \\ \text{et } \pi(\underline{r}^0) = (1-\beta)^n\end{gathered}$$

with :

β = the probability to have a change.

Let $\lambda = \frac{1-\beta}{\beta}$, we have [12] :

$$\begin{gathered}\sup_{\{\underline{r}^1,\underline{r}^0\}} \{\ln(h\,(\underline{y}/\underline{r}^1;\underline{\theta}^1)) - \ln(h\,(\underline{y}/\underline{r}^0;\underline{\theta}^0))\} \\ \lessgtr^{H_0}_{H_1} \ln\lambda\end{gathered} \tag{10}$$

In the case of J signals, the MAP criterion of the change detection in the statistical distribution of J-dimensional process is given by :

$$\begin{gathered}\sup_{\{\underline{r}^1,\underline{r}^0\}} \{\textstyle\sum_{j=1}^{J} (\ln(h_j(\underline{y}_j/\underline{r}^1_j;\underline{\theta}^1_j)) - \ln(h_j\,(\underline{y}_j/\underline{r}^0_j;\underline{\theta}^0_j)))\} \\ \lessgtr^{H_0}_{H_1} \ln\lambda\end{gathered} \tag{11}$$

,where $\underline{r}^i_j$ is the configuration of change point associated to the hypothesis H^i on the process j and $\underline{y}_j$ the process j.

4 Hybrid fusion model

The hybrid fusion method we propose combines a centralized fusion criterion of detection with a decentralized one. The centralized fusion MAP criterion is defined as :

$$\begin{gathered}\sup_{\{\underline{r}^1,\underline{r}^0\}} \{\textstyle\sum_{j=1}^{J} (\ln(h_j(\underline{y}_j/\underline{r}^1_j;\underline{\theta}^1_j)) - \ln(h_j\,(\underline{y}_j/\underline{r}^0_j;\underline{\theta}^0_j)))\} \\ \lessgtr^{H_0}_{H_1} \ln\lambda\end{gathered} \tag{12}$$

The distributed fusion MAP criterion is defined as :

$$\begin{gathered}\textstyle\sum_{j=1}^{J} \{\sup_{\{\underline{r}^1,\underline{r}^0\}} (\ln(h_j(\underline{y}_j/\underline{r}^1_j;\underline{\theta}^1_j)) - \ln(h_j\,(\underline{y}_j/\underline{r}^0_j;\underline{\theta}^0_j)))\} \\ \lessgtr^{H_0}_{H_1} \ln\lambda\end{gathered} \tag{13}$$

When a change occurs at the same time on the processes the centralized fusion criterion will offer the best performances. Indeed in this case the informations about change on the two sensors are synchronized. When the change on the signals occurs at different time the distributed MAP criterion gives in this case the best results. Then the problem is the definition of an hybrid system that fused the centralised and distributed fusion method. The goal of the fusion will be always to have the best performances of the two methods.

In this context we want to define the decision rule of each fusion method and the global fusion rule that combines this decisions. The thresholds of the centralised and distributed fusion method and the rule of combination are choosing to maximize the Neyman Pearson criterion. To define the decision rule of each fusion method that maximizes the Neyman Pearson criterion, we use an optimization element by element. Then we search the decision rules that maximize the Lagrangian L [5], written :

$$L = P_D - \lambda(P_F - \alpha) \tag{14}$$

Let consider here the case of N local detector and a global fusion detector that combine the local decisions. $u_i = j$ is the decision of the hypothesis H_j by the detector i. $u_0 = j$ is the global decision of the fusion system. We have :

$$P_f = P(u_0 = 1/H_0) = \sum_u P(u_0 = 1/u)P(u/H_0) \tag{15}$$

,where u is all the possible combinations of detections. Then we have :

$$L = P_D - \lambda(P_F - \alpha) = P_D - \lambda P_F + \alpha\lambda \tag{16}$$

with :

$$L = \sum_{u=1}^{N} P(u_0 = 1/u)P(u/H_1) - \tag{17}$$
$$\lambda((\sum_u P(u_0 = 1/u)P(u/H_0)) - \alpha)$$

and :

$$L = \lambda\alpha + \sum_u P(u_0 = 1/u)[P(u/H_1) - \lambda P(u/H_0)] \tag{18}$$

We use an optimization elements by elements, and we have:

$$\begin{aligned} L &= \lambda\alpha \\ &+ \sum_{u^k} P(u_0 = 1/u_k = 0, u^k). \\ &\quad [P(u_k = 0, u^k/H_1) - \lambda P(u_k = 0, u^k/H_0)] \\ &+ \sum_{u^k} P(u_0 = 1/u_k = 1, u^k). \\ &\quad [P(u_k = 1, u^k/H_1) - \lambda P(u_k = 1, u^k/H_0)] \end{aligned} \tag{19}$$

where, $u^k = (u_1, \cdots, u_{k-1}, u_{k+1}, \cdots u_N)^T$. When the measurements are correlated like in our case of fusion of different criterion of decision, we have :

$$\begin{aligned} &P(u_1, u_2, \cdots, u_N/H_1) \\ &= P(u_1/H_1) \prod_{j=2}^{N} P(u_j/u_1, u_2, \cdots, u_{j-1}, H_1) \end{aligned} \tag{20}$$

with N the number of sensors.
Therefore :

$$\begin{aligned} &P(u_k = 0, u^k/H_1) \\ &= P(u_1/H_1) \prod_{j=2, j\neq k}^{N} [P(u_j/u_1, \cdots, u_{j-1}, H_1) \\ &\quad .P(u_k = 0/u_1, \cdots, u_{k-1} H_1)] \end{aligned} \tag{21}$$

Let :

$$\begin{aligned} &P'(u^k/H_1) \\ &= P(u_1/H_1) \prod_{j=2, j\neq k}^{N} P(u_j/u_1, \cdots, u_{j-1}, H_1) \\ &\neq P(u^k/H_1) \end{aligned} \tag{22}$$

Since u_k doesn't depend on H_i, we can write :

$$\begin{aligned} &P(u_k = 1/u_1, \cdots, u_{k-1}, H_i) \\ &= \int_{y_k} P(u_k = 1/u_1, \cdots, u_{k-1}, y_k)P(y_k/H_i)dy_k \end{aligned} \tag{23}$$

We can write :

$$\begin{aligned} &L = C^k \\ &+ \int_{y_k} P(u_k = 1/u_1, \cdots, u_{k-1}, y_k)[C_1^k P(y_k/H_1) \\ &-\lambda C_0^k P(y_k/H_0)]dy_k \end{aligned} \tag{24}$$

with :

$$\begin{aligned} &C^k = \lambda\alpha \\ &+ \sum_{u^k} P(u_0 = 1/u_k = 0, u^k)[P'(u^k/H_1) \\ &-\lambda P'(u^k/H_0)] \end{aligned} \tag{25}$$

and :

$$\begin{aligned} &C_i^k = \sum_{u^k}[P(u_0 = 1/u_k = 1, u^k) \\ &-P(u_0 = 1/u_k = 0, u^k)]P'(u^k/H_1) \end{aligned} \tag{26}$$

C^k is independent of the decision rule associated to the detector k, then L is maximum when the integral of the expression 24 is maximum. We have :

$$\begin{aligned} &P(u_k = 1/u_1, \cdots, u_{k-1}, y_k) = 0, if \\ &C_1^k P(y_k/H_1) - \lambda C_0^k P(y_k/H_0) < 0 \end{aligned} \tag{27}$$

and :

$$\begin{aligned} &P(u_k = 1/u_1, \cdots, u_{k-1}, y_k) = 1, if \\ &C_1^k P(y_k/H_1) - \lambda C_0^k P(y_k/H_0) > 0 \end{aligned} \tag{28}$$

The decision rule of the detector k is given by:

$$\frac{P(y_k/H_1)}{P(y_k/H_0)} \lesssim_{H_1}^{H_0} \lambda_k \tag{29}$$

With :

$$\lambda_k = \lambda \frac{C_0^k}{C_1^k} \tag{30}$$

For example, we consider here the case of our application, two detectors (the decisions are coming from the two fusion methods) and the fusion rule which is a AND combination. Let λ_1 be the threshold value for the centralized fusion and λ_2 the threshold value for the distributed fusion criterion :

$$\lambda_1 = \lambda \frac{Pf_2}{Pd_2} \tag{31}$$

and :

$$\lambda_2 = \lambda \frac{Pf_1^1}{Pd_1^1} \tag{32}$$

Where Pd_1^1 is the probability to detect with the centralized fusion method when we have detected with the decentralized fusion method.

5 Experimentations

5.1 Off-line performances

The off-line segmentation consists in detecting the change in the signal when all the data are available. We measure the performances of the system on a fixed signal of known characteristics. The probability of detection (Pd) and false-alarm (Pfa) are calculated here by simulation on 5000 realisations of the processes. The AND fusion rule, that gives better results, is used in our application.

We show Figure 3 the segmentation results for the fusion of two processes having a simultaneous change at position 40 on 50 samples. The signals have a change of 1 and 1.5 in their mean and an additive Gaussian noise of variance 25. We present the ROC curve (Pd=f(Pfa)) of the detection obtained for various threshold values. In this figure the ROC curve is given for the centralized fusion method, the classical MAP detection on L1 and L2 and the hybrid method we propose.

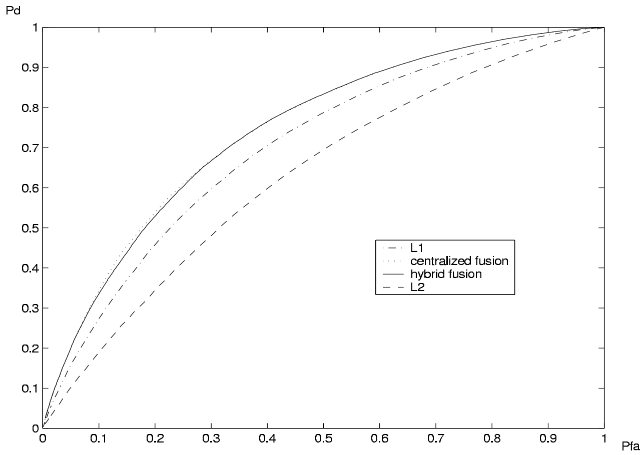

Fig. 3: ROC curve of test signals

On figure 4 we represent the evolution of the probability of detection for a fixed probability of false-alarm (Pfa = 0.3) according to the increasing distance between the ruptures of the two signals.

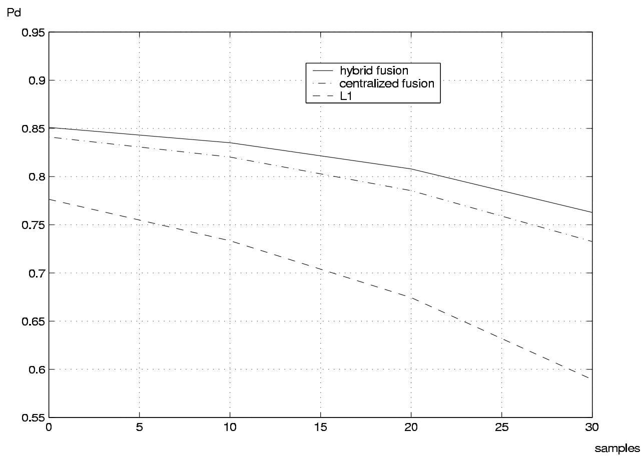

Fig. 4: Probability of detection of test signals

On figures 5 and 6 are presented the same results as in figures 3 and 4 for a synthetic GPS signal with a power of 51 dB-Hz on L1 and 45 dB-Hz on L2. In this experimentation, the sampling rate is 40.92 MHz and we detect the shift of one sample of code.

We can notice on figure 3 and figure 5 that we obtain the same performances for the proposed method (hybrid fusion) and the optimal centralized fusion method when the data are synchronized.

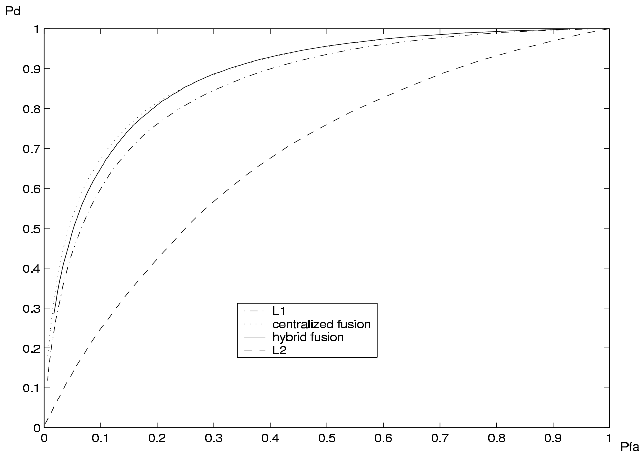

Fig. 5: ROC curve of GPS signals

We show figure 4 and figure 6 that the performances of the proposed method are higher than the traditional optimal method when the ruptures are not simultaneous any more.

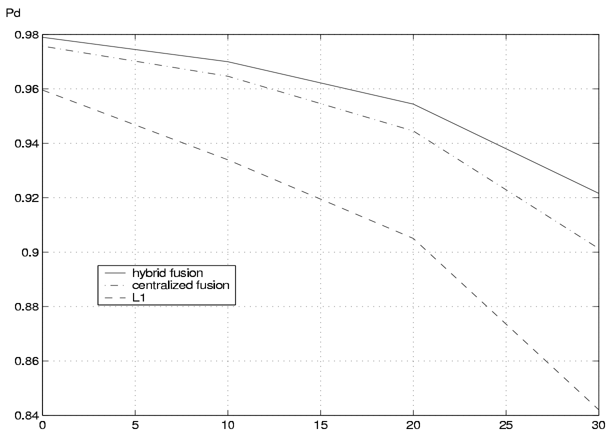

Fig. 6: Probability of detection of GPS signals

5.2 Application to the GPS signal

In this experimentation we simulate the tracking of the code on the carrier frequencies L1 and L2, when there is a relative movement between the receiver and a satellite. We consider a GPS signal with a power of 51 dB-Hz on L1 and 45 dB-Hz on L2 and a sampling rate of 40.92 MHz at the receiver.

Table 1: Square mean error.

Shifts	-8	-2	+2	+8
L1	2.5042	2.5042	2.5042	2.5042
centralized	2.2675	2.3016	2.3444	2.4088
hybrid	2.2481	2.2766	2.3290	2.3516

We represent figure 7 the evolution of the theoretical discriminator (in full line) of the two signals. We can notice the introduction of a shift between the two signals. In dotted lines is represented an example of tracking realized for a window of 20 samples for a Pf = 0.05.

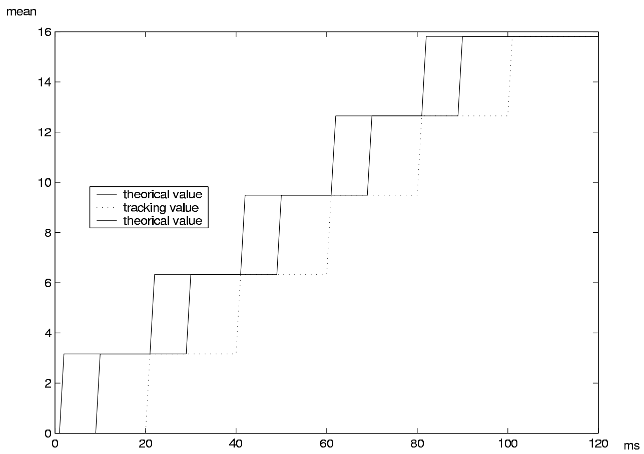

Fig. 7: Example of tracking dicriminator

We report table 1 the mean-square error between the theoretical discriminator of L1 and the tracking results. This error is given for various values of shifts between the discriminator of L1 and L2. We can notice that the results obtained by the proposed method are the best for this experimentation.

6 Conclusion

In this article we propose to fuse the GPS code tracking achieved on multi carrier frequencies. The code tracking is based on the shift code detection, provoked by the movement of the receiver and the satellite. The hybrid fusion algorithm we proposed fuses the code detection on multi carrier frequencies when the shifts are no necessary simultaneous. In this article we define the MAP detection criterion of the shift instants and the thresholds in the Neyman Pearson sense for the fusion. We show in the experimentation on synthetic GPS L1 and L2 signals, that we have better results with our method than in the case of the classical centralised algorithm. The perspectives of this work are about the generalization of the method for more than two carrier frequencies and its application to the future signals GPS L5 and GALILEO.

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