Viterbi Data Association Tracking using Amplitude Information

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Abstract – *The problem of tracking under adverse conditions, such as when the target SNR is low, is known to be a difficult one. One approach to handling this problem is to use a sophisticated tracking method based on the multi-hypothesis tracking (MHT) algorithm. An alternative is to incorporate additional information such as the strength of the measured return. Another option is to combine the two approaches. This paper investigates the performance of a tracker from the MHT family when tracking targets at low SNR, both with and without the use of signal amplitude information.*

Keywords: target tracking, track maintenance, Viterbi data association, low SNR

1 Introduction

Under benign conditions, such as when the target signal to noise ratio is high and the density of clutter is low, it is not difficult to obtain good tracking performance. However, when the target SNR is low the tracker must be capable of handling situations when the probability of detection is low or there are large amounts of clutter or both. This paper investigates the ability of a tracking algorithm based on the Viterbi algorithm to handle such adverse conditions.

The Viterbi algorithm [1] is a dynamic programming technique for finding the shortest path through a trellis in a computationally efficient manner. The use of the Viterbi algorithm for solving the data association problem when tracking targets in clutter was first suggested by Quach and Farooq [2]. Their approach finds the approximate maximum likelihood assignment of measurements to targets. The problem they considered was that of tracking a single, manoeuvring target that was always present in the surveillance region. This method was modified in [3] to suit the problem of tracking multiple, non-manoeuvring targets in heavy clutter. The method of [3] also incorporated a target existence model for automatic track maintenance in scenarios where targets may enter and exit the surveillance region at arbitrary times. These papers showed that Viterbi Data Association (VDA) trackers are more effective at tracking than traditional Probabilistic Data Association (PDA) style trackers [4, 5] when the probability of target detection is low or there is heavy clutter.

In [6, 7], Lerro and Bar-Shalom examined methods for improving track formation and track maintenance performance for PDA trackers for both manoeuvring and non-manoeuvring targets when the probability of detection is low. They showed that making use of the strength of target returns as well as kinematic information in the tracking filter decreased both track loss and track initiation delay. Similar conclusion were drawn for the use of signal strength information in a traditional MHT tracker in [8].

This paper presents an extension of the VDA tracker of [3] that includes signal strength information for tracking a single, non-manoeuvring target in clutter at low SNRs. It quantifies the improvement in track formation and track maintenance statistics this provides over the standard VDA tracker.

The next section describes the target kinematic and signal strength models. Section 3 derives the single target VDA tracker with amplitude information. The track formation and maintenance metrics are defined in Section 4, while the simulation results are given in Section 5.

2 Target Motion and Signal Models

Consider the problem of tracking a single target in clutter. The target dynamics are modelled in discrete time in the standard manner [5] using a state-space model of the form

$$x(k+1) = F(k)x(k) + w(k) \quad (1)$$
$$y(k) = H(k)x(k) + v(k) \quad (2)$$

where $x(k)$ is the target state, $y(k)$ is the measurement of the target kinematics and $\{w(k)\}$ and $\{v(k)\}$ are zero mean, white Gaussian noise processes with known covariances $Q(k)$ and $R(k)$ respectively. Given that there is a non-zero probability of a false alarm, in general more than one measurement may be received in a given scan. Let $Z(k) = \{z_1(k), z_2(k), \ldots, z_{m(k)}(k)\}$ be the set of measurements at time k. The probability of detecting the target is denoted by P_D and $P_D < 1$. At most, one of the measurements in the set $Z(k)$ is assumed to be from the target at scan k. For simplicity, clutter is assumed to be uniformly distributed in the surveillance region, although more elaborate clutter models can be incorporated into the tracker structure if desired.

The amplitude component of the measurement vector is the output of a bandpass matched filter followed by an envelope detector. The target is assumed to be a Swerling

1 target. That is, the target return is a slowly fluctuating amplitude modulated narrowband signal in the presence of narrowband Gaussian noise. Assuming normalised background noise, the probability densities of the signal envelope for noise only and when the target is present may be written as [9]

$$p_0(s) = s\exp\left(\frac{-s^2}{2}\right), \; s \geq 0 \quad (3)$$

$$p_1(s) = \left(\frac{s}{1+A}\right)\exp\left(\frac{-s^2}{2(1+A)}\right), \; s \geq 0 \quad (4)$$

where A is the expected SNR of the target returns. Given a detection threshold of τ the probability densities of the detections are

$$p_0^\tau(s) = \frac{1}{P_{FA}} p_0(s), \; s \geq \tau \quad (5)$$

$$p_1^\tau(s) = \frac{1}{P_D} p_1(s), \; s \geq \tau \quad (6)$$

where $P_{FA} = \int_\tau^\infty p_0(s)ds$ and $P_D = \int_\tau^\infty p_1(s)ds$.

3 VDA Tracker with Amplitude Information

3.1 Maximum Likelihood Data Association

The VDA tracker of [3] is designed to find the approximate maximum likelihood association of measurements to the target, when the target may enter or leave the surveillance region at arbitrary times. This maximum likelihood data association problem can be expressed as follows. Given a set of $m(k)$ measurements at time k the set of all possible events $\{\theta_j(k)\}$ can be written as

- $\theta_{-1}(k) =$ the target is not in the surveillance region
- $\theta_0(k) =$ the target is in the surveillance region but is not detected
- $\theta_j(k) =$ the target is in the region, is detected and $z_j(k)$ is the target measurement, $j = 1,\ldots,m(k)$.

Given a sequence of scans from $k = 1,\ldots,N$, the data association problem is then to find $\Gamma = \{\theta_{j_1^*},\ldots,\theta_{j_N^*}\}$ that satisfies

$$\max_\Gamma p(Z(1),\ldots,Z(N),\theta_{j_1},\ldots,\theta_{j_N}|x(1),\ldots,x(N)) \quad (7)$$

Let $Z^k = \{Z(1),\ldots,Z(k)\}$ and $X^k = \{x(1),\ldots,x(k)\}$ and consider an arbitrary sequence of associations ending with the association $\theta_j(k)$, written as $\Theta_j(k)$. Defining

$$\bar{d}_j(k) \stackrel{\triangle}{=} \max_{j_1,\ldots,j_{k-1}} p(Z^k,\Theta_j(k)|X^k), \;\; j = -1,0,\ldots,m(k) \quad (8)$$

it can be shown [10] that

$$\bar{d}_j(k) = \max_{i=-1,\ldots,m(k-1)} \bar{a}_{ij}(k)\bar{d}_i(k-1) \quad (9)$$

where

$$\bar{a}_{ij}(k) \stackrel{\triangle}{=} p(Z(k),\theta_j(k)|Z^{k-1},\Theta_i(k-1),X^k) \quad (10)$$

Taking the negative logarithm of (9) reduces the maximum likelihood data association problem to an additive minimisation problem.

Consider a trellis consisting of $m(k)+2$ nodes at time k, where each node represents a possible association event $\theta_j(k), j = -1,\ldots,m(k)$. Let the transition cost from node $\theta_i(k-1)$ to $\theta_j(k)$ be given by

$$a_{ij}(k) \stackrel{\triangle}{=} -\ln \bar{a}_{ij}(k). \quad (11)$$

The Viterbi algorithm [1] can then be used to find the minimum cost path through the trellis in a computationally efficient manner. Given the definition of the transitions costs in (11), this path corresponds to solution of the maximum likelihood data association problem (7).

To calculate the transition costs (11) exactly it would be necessary to know the true state sequence $\{X^k\}$. Since this is not available, the VDA tracker makes use of the estimated state sequence $\{\hat{X}^k\}$ obtained by running a Kalman filter along each path through the trellis. Thus the VDA trackers of both [2] and [3] are approximate maximum likelihood tracking algorithms.

3.2 Transition Costs

To implement a VDA tracker using amplitude information (VDA-AI) we need to define expressions for the transitions costs. First, let the probability the target is in the surveillance region at time k, $E(k)$, evolve according to a two state Markov chain [11]

$$Pr(E(k)|E(k-1)) = \delta_0 \quad (12)$$

$$Pr(E(k)|\bar{E}(k-1)) = \delta_1 \quad (13)$$

then the *a priori* probability of track existence at time k is given by

$$P_E(k|k-1) = \delta_0 P_E(k-1|k-1) + \delta_1\{1 - P_E(k-1|k-1)\} \quad (14)$$

Given the target model of Section 2, and assuming a diffuse prior for the expected number of clutter points in the gate, the transitions costs for the VDA-AI tracker are given by the equations

$$\bar{a}_{i,-1}(k) = \Delta_i^{-1}\frac{1}{V_i(k)}(1 - P_E(k|k-1)) \quad (15)$$

$$\bar{a}_{i,0}(k) = \Delta_i^{-1}\frac{1}{V_i(k)}(1 - P_D P_G)P_E(k|k-1) \quad (16)$$

$$\bar{a}_{ij}(k) \approx \Delta_i^{-1}\frac{P_D P_E(k|k-1)}{m_i(k)} \times \quad (17)$$

$$\mathcal{N}(\nu_j(k);0,S_i(k|k-1))\frac{p_1^\tau(s_j(k))}{p_0^\tau(s_j(k))} \quad (18)$$

where Δ_i is a normalisation constant and

- $V_i(k)$ is the volume of the validation gate at k for the track ending with the association $\theta_i(k-1)$;
- P_G is the probability the target measurement is in the gate;

- $m_i(k)$ is the number of measurements in the gate;
- $\nu_i(k) = y(k) - \hat{y}_i(k|k-1)$ where $\hat{y}_i(k|k-1)$ is the predicted value of $y(k)$ given the track ending with the association $\theta_i(k-1)$;
- $\mathcal{N}(\cdot;\mu,\Sigma^2)$ is a multivariate Gaussian PDF with mean μ and covariance Σ^2;
- $S_i(k|k-1)$ is the predicted measurement error covariance from the Kalman filter; and
- $s_j(k)$ is the measured amplitude of the j-th detection at time k.

The transition costs in the standard VDA tracker, without amplitude information, are given by the equations [3]

$$\bar{a}_{i,-1}(k) = \Lambda_i^{-1}\lambda(1 - P_E(k|k-1)) \quad (19)$$
$$\bar{a}_{i,0}(k) = \Lambda_i^{-1}\lambda(1 - P_D P_G)P_E(k|k-1) \quad (20)$$
$$\bar{a}_{ij}(k) \approx \Lambda_i^{-1}P_D P_E(k|k-1) \times \quad (21)$$
$$\mathcal{N}(\nu_j(k);0,S_i(k|k-1)) \quad (22)$$

where λ is the expected clutter density per unit volume. The equations for the standard VDA tracker are derived under the assumption that the number of clutter points in the surveillance region follows a Poisson distribution.

The normalisation constants for the VDA tracker, Λ_i, and the VDA-AI tracker, Δ_i, can be calculated by noting that

$$\sum_{j=-1}^{m(k)} Pr(\theta_j(k)|Y^{k-1},\Theta_i(k-1)) = 1 \quad (23)$$

4 Track Formation and Maintenance Metrics

The following set of metrics will be used to evaluate the performance of the VDA trackers, with and without the use of amplitude information.

- *Track Initiation Delay* : this is the number of scans between the first target detection in the surveillance region and the time a track is initiated on the target. Only tracks that are ultimately confirmed will be used to calculate this metric. Thus a track that is initiated on the target but is deleted before being confirmed will not be considered when calculating this metric. Instead, such a track will contribute to the number of missed targets metric.
- *Track Confirmation Delay* : this is the number of scans between a track being initiated on the target and that track being confirmed.
- *Track Overshoot* : this is the number of scans a confirmed true target track is continued after the target has left the surveillance region.
- *Number of False Tracks* : this is the number of confirmed tracks that are not true target tracks.
- *Number of True Tracks* : this is the number of tracks that are classified as true target tracks at the time they are confirmed. A track is classified as a true track if it make use of at least half of the available target detections from the scan it was initiated until the scan it was confirmed.
- *Number of Lost Tracks* : this is the number of confirmed true target tracks that lose the target before the target leaves the surveillance region. A track is classed as having lost the target once the true target measurement is no longer used by the track for the remainder of the track's lifetime.
- *Number of Missed Targets* : this is the number of times there is no confirmed true target track.

Tracks are confirmed and deleted using the track confidence measure defined as follows. Let the confidence for a track ending with the association $\theta_j(k)$ be defined as

$$c_j(k) \stackrel{\triangle}{=} Pr(\Theta_j(k)|Z^k,X^k) \quad (24)$$

which can be re-expressed using Bayes' rule as

$$c_j(k) = \frac{p(Z^k,\Theta_j(k)|X^k)}{p(Z^k|X^k)} \quad (25)$$
$$= \frac{\bar{d}_j(k)}{\sum_{r=-1}^{m(k)} \bar{d}_r(k)} \quad (26)$$

The *a posteriori* probability of track existence is then $P_E(k|k) = 1 - c_{-1}(k)$. This measure is compared against confirmation and deletion thresholds, P_{con} and P_{del}, to confirm or delete tracks.

5 Simulations

The simulations are based on a target moving in a two dimensional space. The target state is the position and velocity in the two Cartesian co-ordinates, while measurements are of position only. The matrices in the state space model (1)–(2) are given by

$$F = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (27)$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (28)$$

$$Q = \begin{bmatrix} \frac{T^3}{3}\sigma_{\dot{x}}^2 & \frac{T^2}{2}\sigma_{\dot{x}}^2 & 0 & 0 \\ \frac{T^2}{2}\sigma_{\dot{x}}^2 & T\sigma_{\dot{x}}^2 & 0 & 0 \\ 0 & 0 & \frac{T^3}{3}\sigma_{\dot{y}}^2 & \frac{T^2}{2}\sigma_{\dot{y}}^2 \\ 0 & 0 & \frac{T^2}{2}\sigma_{\dot{y}}^2 & T\sigma_{\dot{y}}^2 \end{bmatrix} \quad (29)$$

$$R = \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix} \quad (30)$$

where $T = 1s$ is the revisit rate. The target position is measured in metres and velocity in m/s. The position measurement error standard deviations are $\sigma_x = \sigma_y = 5m$. The standard deviations for the state process noise are $\sigma_{\dot{x}} = \sigma_{\dot{y}} = 0.1m/s$.

The surveillance region consisted of 10,000 cells of resolution $10 \times 10m$. Clutter measurements were generated uniformly over the entire surveillance region at each time k. The amplitudes of these false measurements were drawn from the Rayleigh distribution given by equation (3). Each simulation ran for 100 scans, with the target present in the region from scan 1 to scan 85. The amplitude of the target measurement was drawn from the Rayleigh distribution given by equation (4). For both the target and clutter, a detection was only declared if the amplitude was above the threshold τ. This threshold was set by specifying the desired probability of a false alarm, P_{FA}, for a range of values ($P_{FA} = 0.001, 0.003, 0.01$). Simulations were run for three different target SNR scenarios: 10dB, 13dB and 20dB.

The VDA-AI tracker transition costs where calculated using the known target SNR. The value for λ in the transition costs of the standard VDA tracker was approximated by the average clutter density per unit volume over the 100 scans.

The parameters for the Markov model for target existence were $\delta_0 = 0.99$ and $\delta_1 = 0.0$. The gate probability was set to $P_G = 0.95$. The track confirmation threshold was $P_{con} = 0.5$ and the deletion threshold $P_{del} = 0.1$. Confirmation and deletion decisions were made based on a running average of the *a posteriori* track existence probability. This averaging process ensures that tentative tracks are propagated for a sufficiently large enough number of scans before a confirmation or deletion decision is made. Without this process, the possibility of the target being missed is greatly increased. However, this is at the expense of a lower bound on the confirmation delay, so the lowest practical value was used in each case. The length of the running average, L, varied between the two types of trackers and also with each level of P_{FA}. The values used are shown in Table 1.

Other than the running average length, the two trackers have the same parameters for all scenarios. No particular attempt was made to optimise the performance of either tracker. Instead, the parameters were chosen so that both trackers would have adequate performance with the same settings so that the effect of the inclusion of amplitude information could be isolated from the effect of varying the tracker parameters.

For each combination of P_D and P_{FA} 200 Monte Carlo simulations were run using the standard VDA tracker and the augmented VDA-AI tracker. The results for the various track initiation and maintenance metrics are shown in Tables 2 and 3. Note, that the sum of the number of true tracks and missed tracks may exceed 200. This indicates that on occasion more than one true target track was present during a single simulation run.

From these results it can be seen that the use of signal strength information significantly reduces the false track rate and, in general, also reduces the track initiation delay. These results are in line with those of [6], where it was shown that use of this information decreased track initiation delay and track loss. In these simulations, the trackers were tuned to have similar track acquisition performance, so that the difference in the track maintenance performance of the two approaches is shown by the false track rate. Alternatively, if the two trackers were turned to have similar false track rates, the missed track and lost track statistics for the VDA-AI tracker would be reduced, as was the case in [6].

6 Conclusions

This paper has shown that the track acquisition and track maintenance performance of the VDA tracker can be improved by incorporating signal strength information into the tracking model. These results are in keeping with those of [6, 7] who showed similar results for PDA trackers and [8] for an MHT-based tracker. The results of all these papers indicate that significant improvements can be made in tracking under adverse conditions for a low additional computational cost by including signal strength information in the existing tracking algorithm, rather than by switching to a more elaborate tracking algorithm.

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Table 1: Track Confidence Running Average Length, L

Scenario	P_{FA}	P_D	SNR (dB)	L	
				VDA	VDA-AI
1	0.001	0.50	10	3	4
2	0.001	0.71	13	3	5
3	0.001	0.93	20	3	6
4	0.003	0.56	10	3	6
5	0.003	0.75	13	4	6
6	0.003	0.94	20	4	6
7	0.01	0.63	10	6	7
8	0.01	0.79	13	6	7
9	0.01	0.95	20	6	7

Table 2: Track Delay Statistics

P_{FA}	SNR (dB)	Initiation Delay		Confirmation Delay		Overshoot	
		VDA	VDA-AI	VDA	VDA-AI	VDA	VDA-AI
0.001	10	20.7	19.3	4.5	5.4	3.9	4.0
0.001	13	12.8	9.3	4.2	6.1	3.1	3.1
0.001	20	4.3	5.3	4.0	5.8	3.2	3.3
0.03	10	33.3	25.2	5.2	7.7	4.0	4.2
0.03	13	21.3	14.6	5.2	7.0	3.2	3.2
0.03	20	15.6	3.2	5.3	6.7	3.2	3.3
0.01	10	34.0	31.3	8.0	9.1	4.0	4.1
0.01	13	29.3	30.3	7.8	8.8	3.3	3.3
0.01	20	22.0	15.4	7.9	8.7	3.2	3.3

Table 3: Track Acquisition and Loss Metrics

P_{FA}	SNR (dB)	True Tracks		False Tracks		Lost Tracks		Missed Tracks	
		VDA	VDA-AI	VDA	VDA-AI	VDA	VDA-AI	VDA	VDA-AI
0.001	10	150	154	59	18	43	46	54	47
0.001	13	206	204	20	0	71	68	1	0
0.001	20	201	200	9	0	21	19	0	0
0.03	10	99	149	162	5	23	29	102	53
0.03	13	155	193	63	8	49	64	45	11
0.03	20	198	201	2	0	18	17	2	0
0.01	10	107	102	379	4	24	16	95	100
0.01	13	155	156	133	2	46	42	47	45
0.01	20	192	199	8	0	23	14	12	2